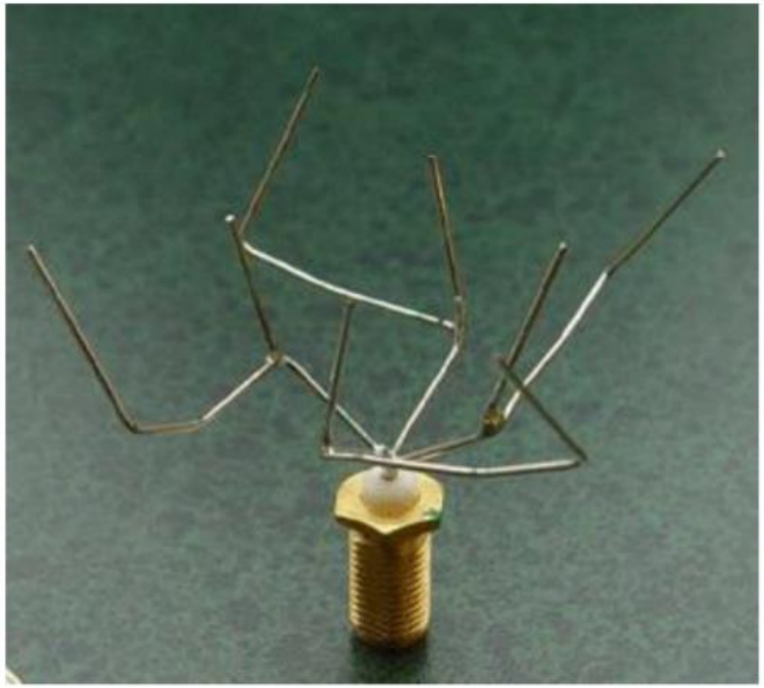
20IS603 Architecture of Intelligent Systems

Optimization Algorithms

Lecture #13

Optimization

- "For a given system, it is the selection of a best element, with regard to some criteria, to achieve optimal results"
- Why use optimization?
  - Finding better (optimal) designs
  - Faster Design Evaluations
  - Useful for trade-off analysis
  - Non-intuitive designs may be found

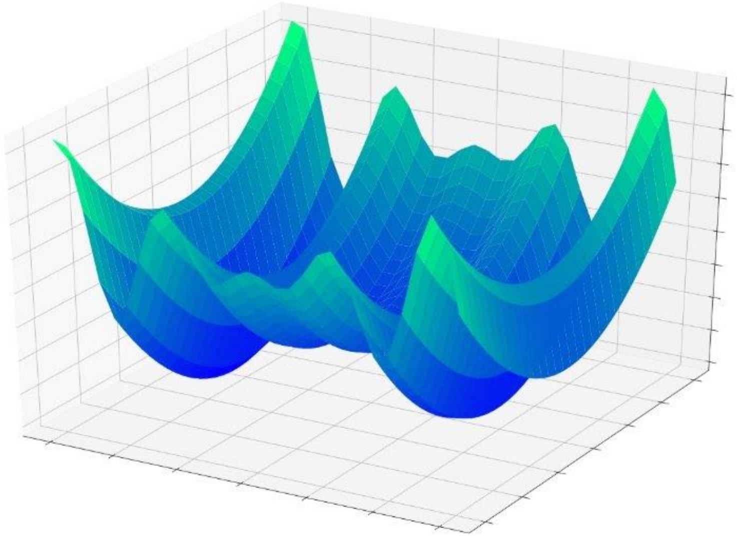

Antenna Design Using Genetic Algorithm

Optimization (2)

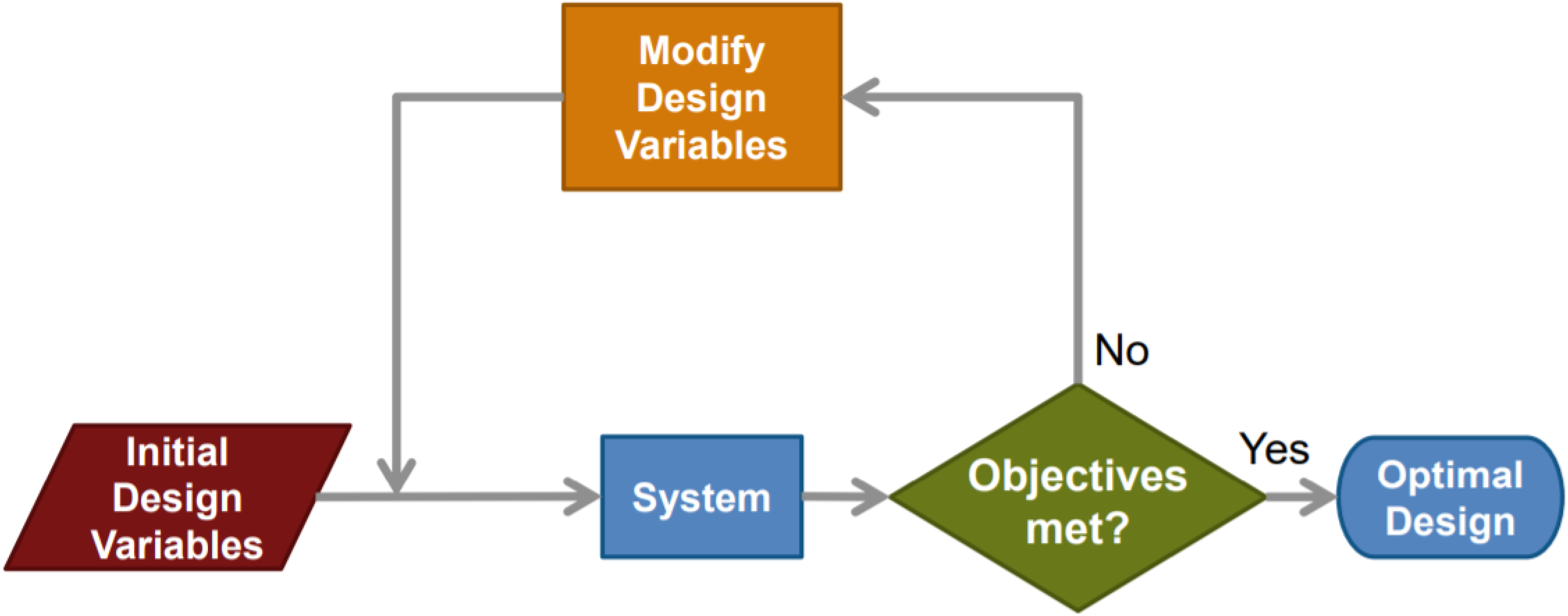
- The process of finding the best values for the variables of a particular problem.
- The goal is to determine the maximum or minimum value of some function of one or more variables.
- Often considered to be one of determining a minimum, and the function that is being minimized is referred to as a *cost function*.
- Cost function is the difference, or error, between a desired output and the actual output.
- Maximizing the value of a function is known as a *fitness function*.
- Fitness is taken to be the negation of the cost and vice versa.

Optimization (3)

- Objective function embraces both fitness and cost.
- Optimization of the objective function mean either minimizing the cost or maximizing the fitness.
- *Deterministic methods* use predictable and repeatable steps, so that the same solution is always found, provided the starting conditions are the same.
- Stochastic methods include an element of randomness that prevents guaranteed repeatability.



Optimization Workflow



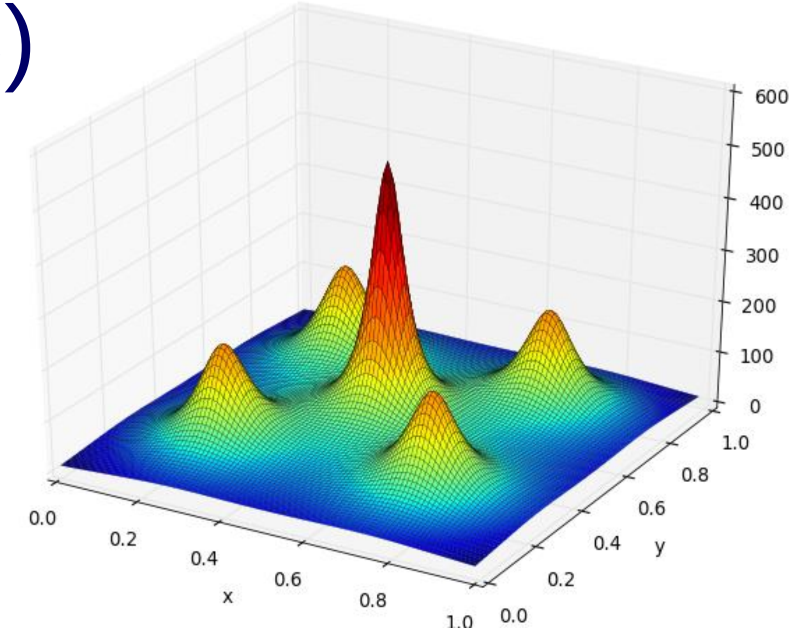
The Search Space

- Problem solving is an important aspect of Intelligent systems.
- *Problem* - consist of a goal and a set of actions that can be taken to lead to the goal.
- *Search* is a method used by computers to examine a problem space in order to find a goal.
- Find the goal as quickly as possible or without using too many resources.
- A *problem space* is considered to be a search space because in order to solve the problem, search the space for a goal.
- The potential solutions to a search problem constitute the *search space* or parameter space.

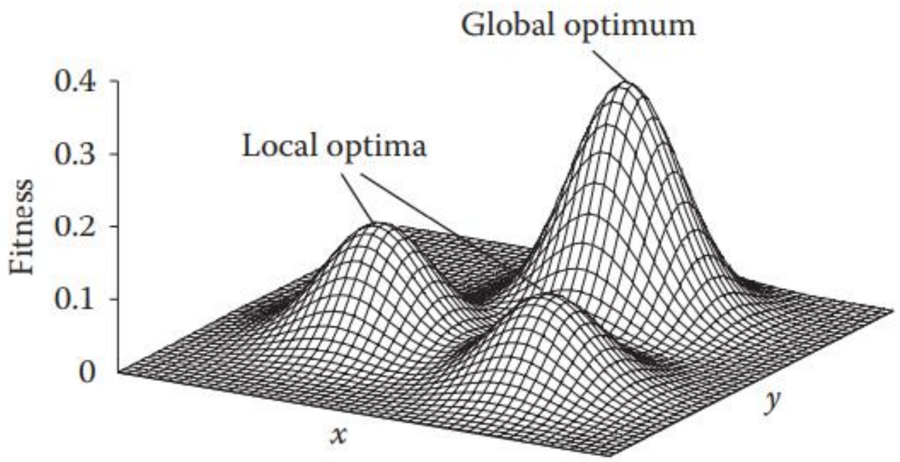
The Search Space (2)

- If a value is sought for a single variable, or parameter, the search space is one-dimensional.
- If simultaneous values of *n* variables are sought, the search space is *n-dimensional*.
- Two main approaches to searching - Data-driven search and goal-driven search
- *Data-driven search* starts from an initial state and uses actions that are allowed to move forward until a goal is reached.
- *Goal-driven search* start at the goal and work back toward a start state, by seeing what moves have led to the goal state.

The Search Space (3)

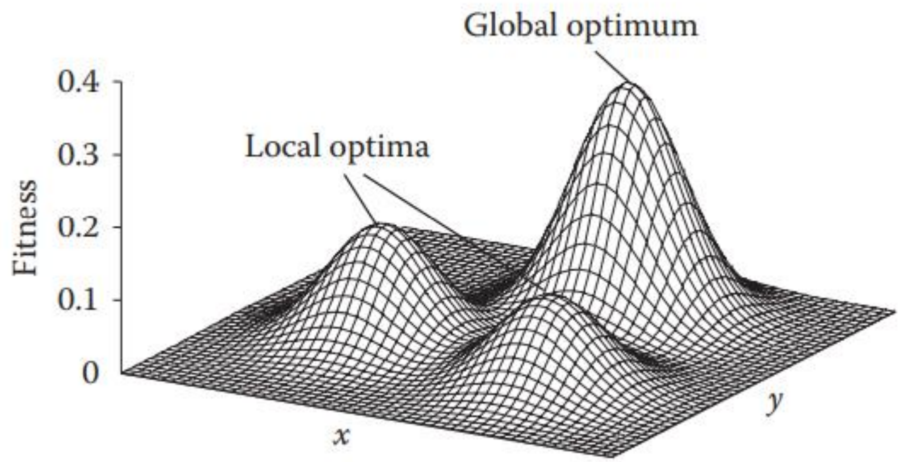


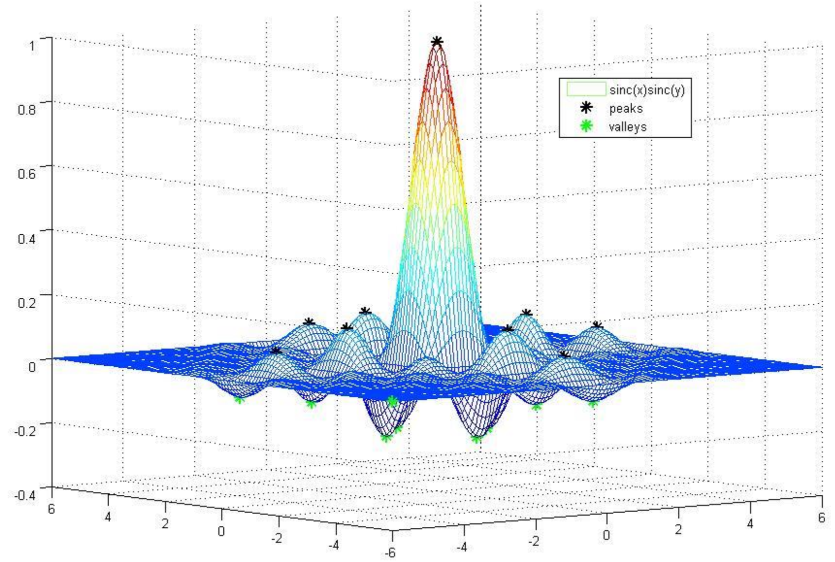
- *Fitness landscape* - plotting the fitness for a two-dimensional search space.
- Optimization algorithm involve finding peaks in the fitness landscape or valleys in the cost landscape.
- A global optimum is the point in the search space with the highest fitness.
- A local optimum is a point whose fitness is higher than all its near neighbors but lower than that of the global optimum
- Regardless of the number of dimensions, there is a risk of finding a local optimum rather than the global optimum for the function.



The Search Space (3)

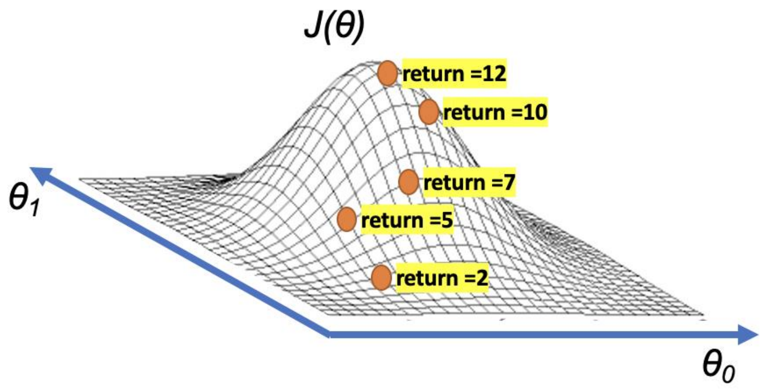
- If neighboring points in the search space have a similar fitness, the landscape is said to be *smooth or correlated*.
- When neighboring points have very different fitnesses, the landscape is said to be *rugged*.
- Fitness of any individual point in the search space is representative of the quality of the surrounding region.

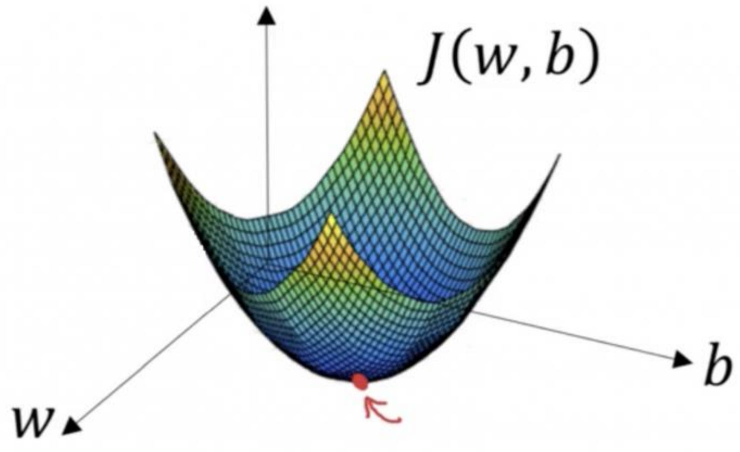




Searching the Parameter Space

- Determining the optimum for an objective function of multiple variables is not straightforward, even when the landscape is static.
- Choosing a starting point and then altering one or more variables in an attempt to increase the fitness or reduce the cost - single "best solution so far" that is refined until no further increase in fitness can be achieved



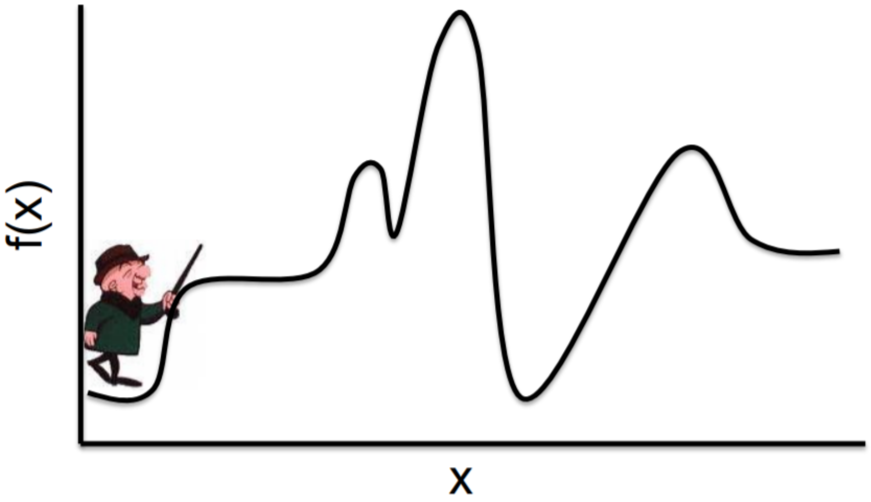


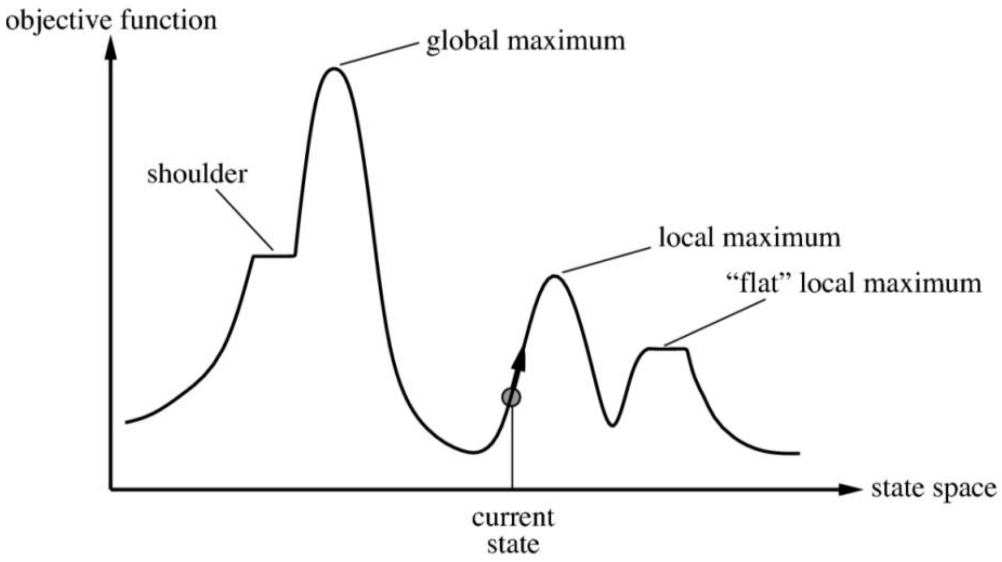
Hill-Climbing Algorithm

- Optimization is viewed as the search for a maximum in a fitness landscape.
- Can equally be applied to a cost landscape - *valley descent*.
- To climb a hill and reach the topmost peak/ point of that hill.
- Based on the heuristic search technique where the person who is climbing up on the hill estimates the direction which leads to the highest peak

Hill-Climbing Algorithm (2)

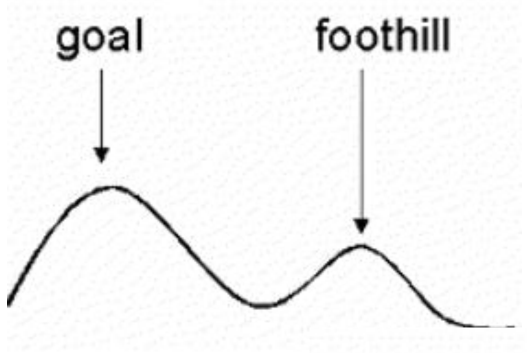
- From a randomly selected start point in the search space (a trial solution), a step is taken in a random direction.
- If the fitness of the new point is greater than the previous position, it is accepted as the new trial solution, otherwise the trial solution is unchanged.
- The process is repeated until the algorithm no longer accepts any steps from the trial solution - at this point the trial solution is assumed to be the optimum.

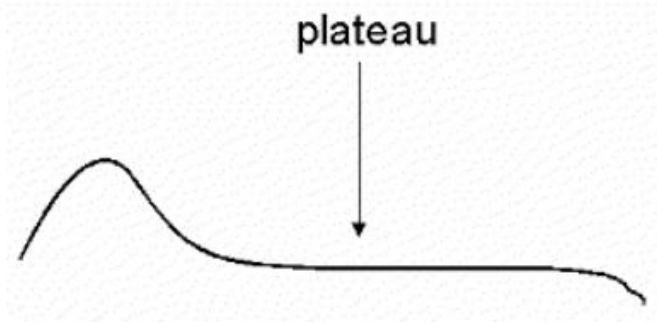


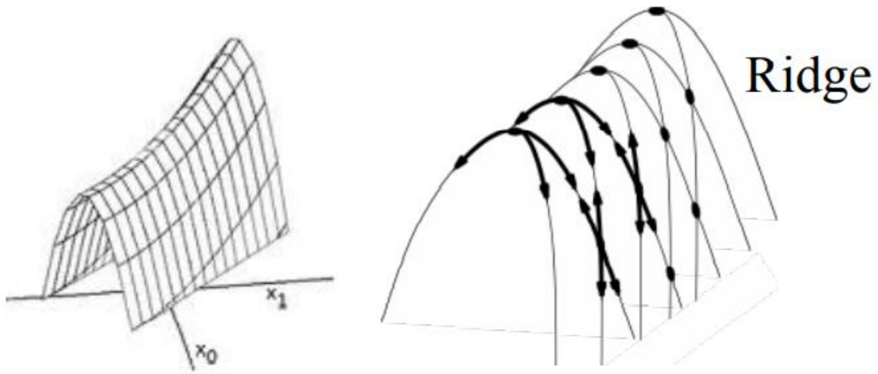


Drawbacks of hill-climbing

- Algorithm is inefficient and offers no protection against finding a local minimum rather than the global one.
- Fooled by foothills, plateaus, and ridges.
- Foothills are often called local maxima.
- Plateaus are an area of the state space where the evaluation function is flat local maximum, from which no uphill path exists.
- Ridges are dropoffs to the sides - A sequence of local maxima - very difficult to navigate for a hill-climbing algorithm

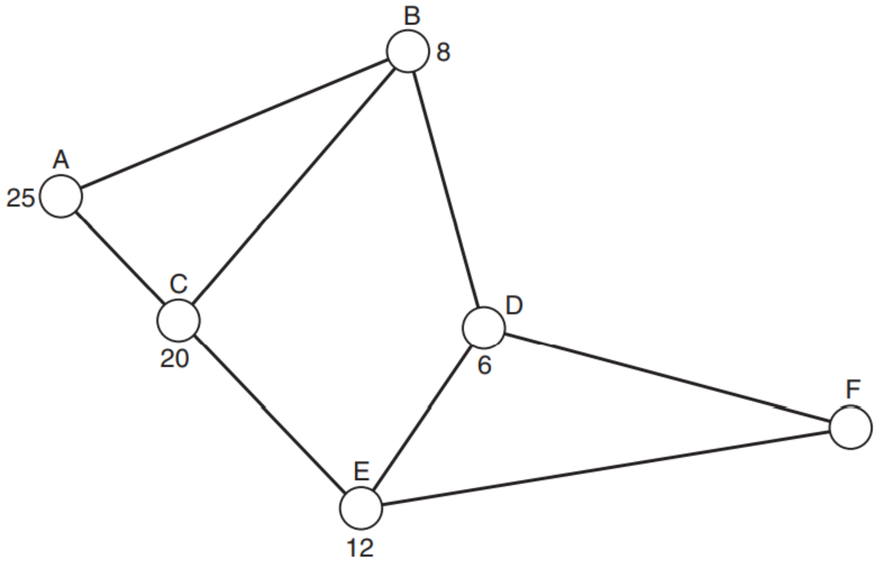






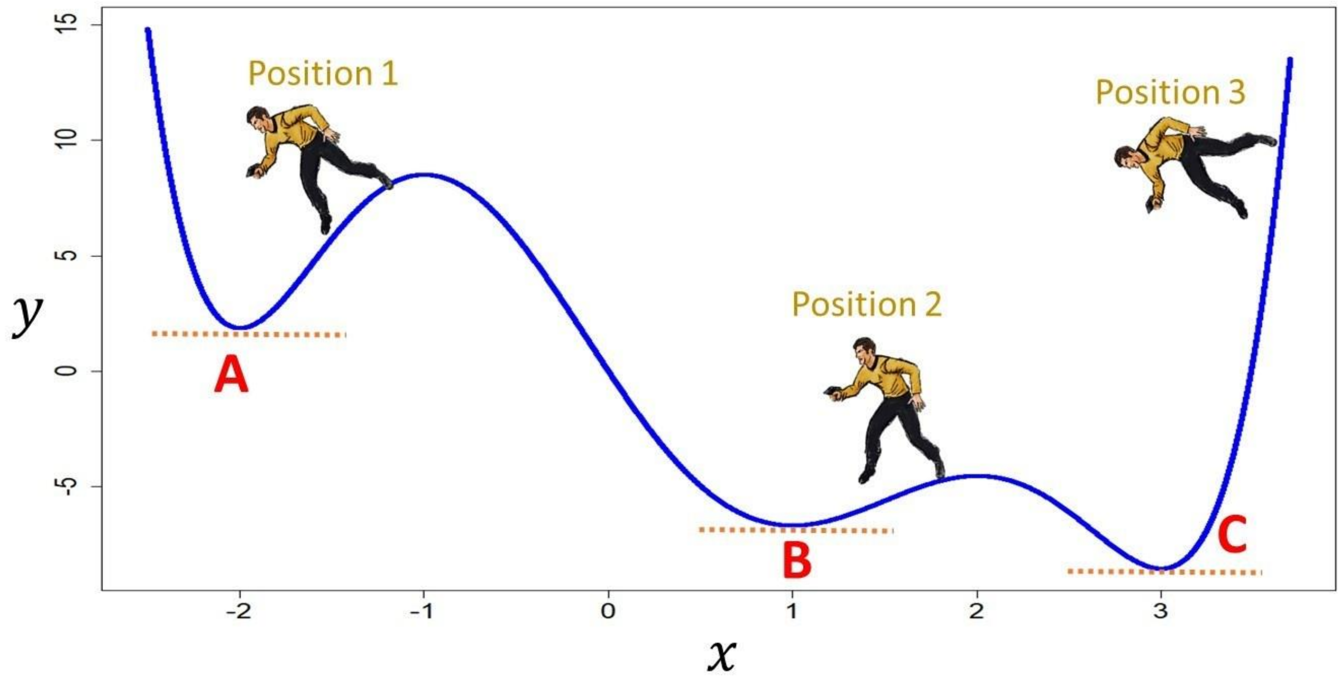
Steepest Gradient Descent or Ascent

- Steepest gradient descent (or ascent) is a refinement of hill climbing that can speed the convergence toward a minimum cost (or maximum fitness).
- From a given starting point, that is, a trial solution, the direction of steepest descent is determined.
- A point lying a small distance along this direction is then taken as the new trial solution.
- The process is repeated until it is no longer possible to descend, at which point it is assumed that the optimum has been reached.
- Slightly more sophisticated than hill climbing, and it offers no protection against finding a local minimum rather than the global one.



Gradient Descent

- Applied in a cost landscape that is continuous and differentiable, that is, where the variables can take any value within the allowed range and the cost varies smoothly.
- Rather than choosing a fixed step size, the size of the steps is allowed to vary in proportion to the local gradient of descent.



Simulated Annealing

- Annealing is a process of producing very strong glass or metal, which involves heating the material to a very high temperature and then allowing it to cool very slowly.
- In the cooling metal, atoms move to form a near-perfect crystal lattice, even though they may have to overcome a localized energy barrier called the activation energy, Ea.
- The atomic rearrangements within the crystal are probabilistic.

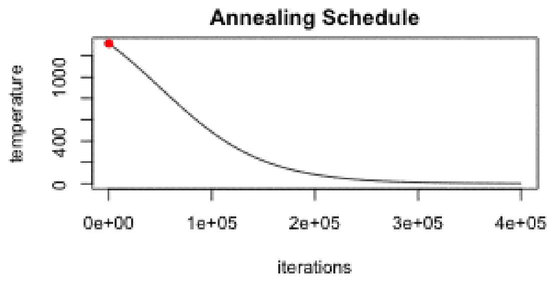
$$P = \exp(-E_a / kT)$$

where k is Boltzmann's constant and T is temperature.

- At high temperatures, the probability approaches 1, while at T = 0 the probability is 0

Simulated Annealing (2)

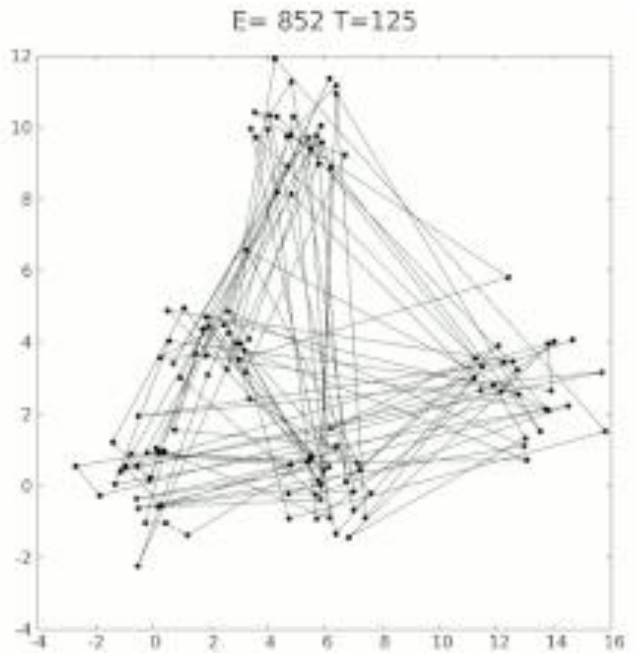
- Local search metaheuristic based method.
- Applied to a multi-value combinatorial problem where values need to be chosen for many variables to produce a particular value for some global function.
- This value is thought of as the energy of the system, and in general the aim of simulated annealing is to find a minimum energy for a system.



Simulated Annealing (3)

- A trial solution is chosen, and the effects of taking a small random step from this position are tested.
- If the step results in a reduction in the cost function, it replaces the previous solution as the current trial solution.
- If it does not result in a cost saving, the solution still has a probability P of being accepted as the new trial solution given by:

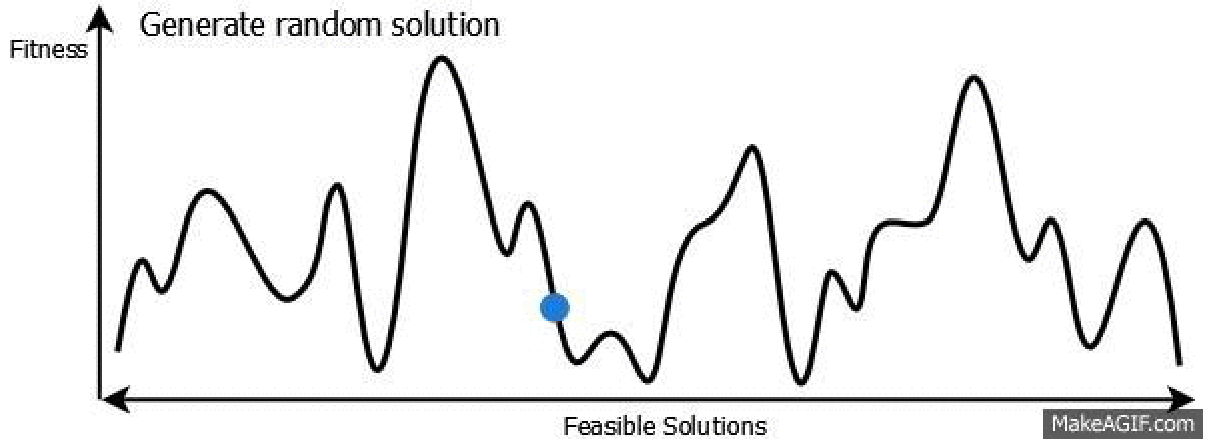
$$P = \exp(-\Delta E / T)$$

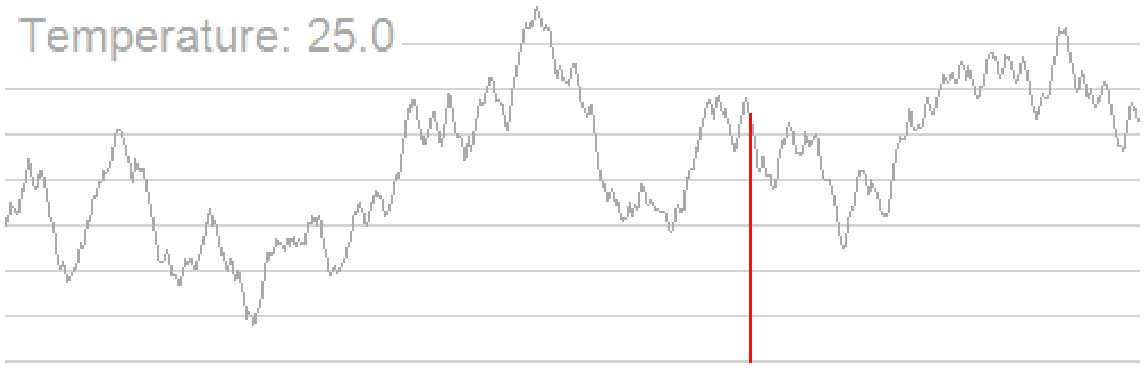


ΔE is the increase in the cost function that would result from the step and is, therefore, analogous to the activation energy in the atomic system.
The temperature T, a numerical value, determines the stability of a trial solution.

Simulated Annealing (4)

- If T is high, new trial solutions will be generated continually.
- If T is low, trial solution will move to a local or global cost minimum.
- The value of T is initially set high and is periodically reduced according to a cooling schedule.
- At the start of the process, T is high; At T = 0, the method is equivalent to the hill-climbing algorithm.
- If the optimization is successful, the final solution will be the global minimum

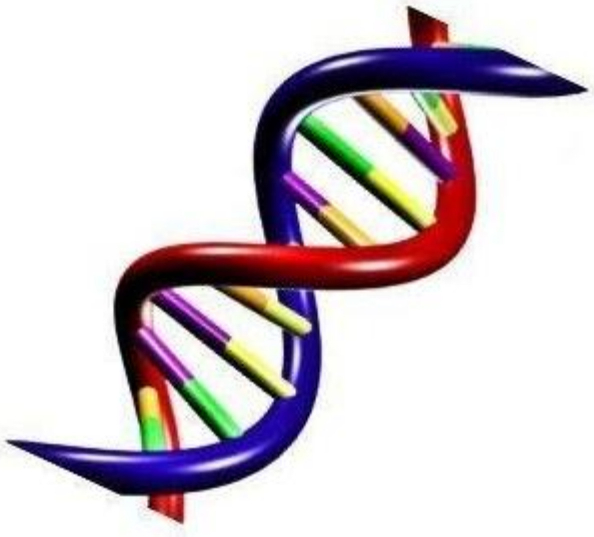




Genetic Algorithms

Genetic Algorithms

- Genetic Algorithms (GAs) are meta-heuristic algorithms inspired by Darwin's theory of natural selection and that belong to a broader class of evolutionary algorithms (EA).
- Process by which successive generations of animals and plants are modified so as to approach an optimum form.
- Each offspring has different features from its parents - it is not a perfect copy.
- If the new characteristics are favorable, the offspring is more likely to flourish and pass its characteristics to the next generation.
- An offspring with unfavorable characteristics is likely to die without reproducing.
- These ideas applied to mathematical optimization, where a population of candidate solutions "evolves" toward an optimum

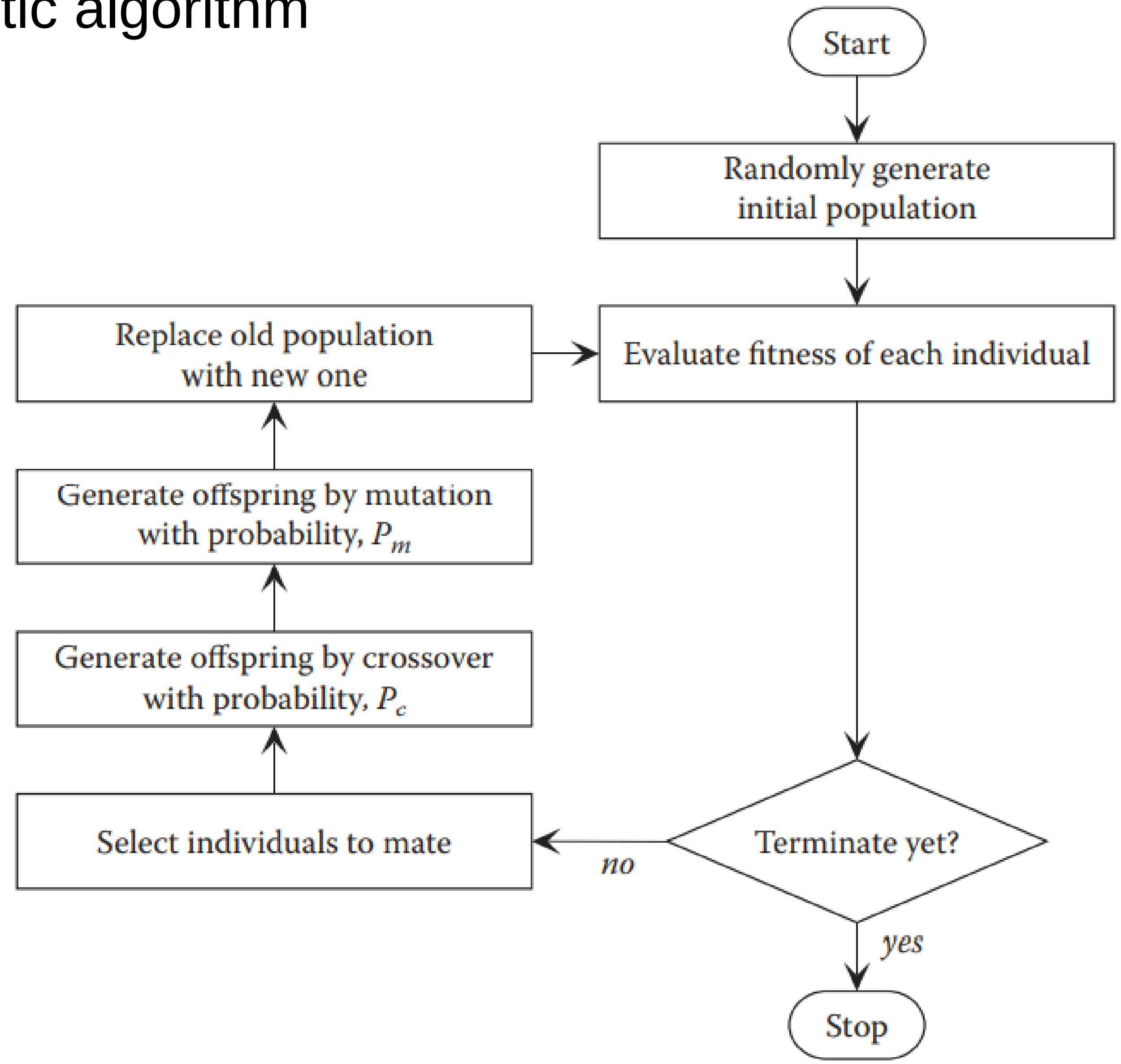


Genetic Algorithms - Terminology

- Each cell of a living organism contains a set of *chromosomes* that define the organism's characteristics.
- The chromosomes are made up of *genes*.
- Each gene determines a particular trait.
- The complete set of genetic material is the *genome*.
- A particular set of gene values constitutes a *genotype*.
- The resulting set of traits is the phenotype.
- Each individual in the population of candidate solutions is graded according to its *fitness*.
- The fittest individuals are selected for reproduction in order to produce offspring of the next generation.
- The higher the fitness of a candidate solution, the greater are its chances of reproducing and passing its characteristics to the next generation.

Phases of GA

- Five phases are considered in a genetic algorithm
  - Initial population
  - Fitness function
  - Selection
  - Crossover
  - Mutation

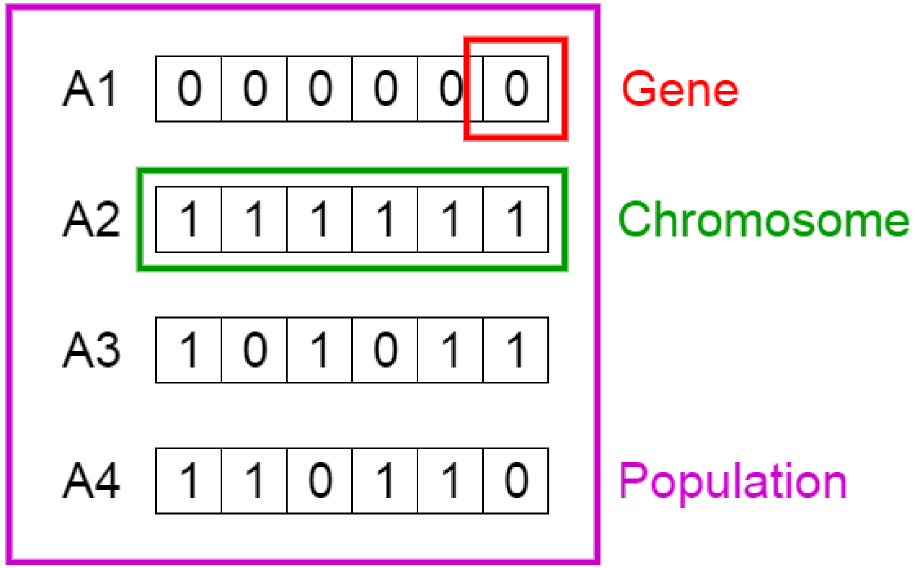


Phases of GA

Initial Population

- The process begins with a set of individuals which is called a Population.
- Each individual is a solution to the problem to be solved.
- An individual is characterized by a set of parameters (variables) known as Genes.
- Genes are joined into a string to form a Chromosome (solution).

- The possible values for the genes are called *alleles*.
- Each position along the chromosome is known as a *locus*.



Phases of GA

Fitness Function

- Individuals are evaluated according to the fitness function.
- Determines how fit an individual is - the ability of an individual to compete with other individuals.
- Gives a fitness score to each individual - probability that an individual will be selected for reproduction is based on its fitness score.

Selection

- To select the fittest individuals and pass their genes to the next generation
- Two pairs of individuals (parents) are selected based on their fitness scores.
- Individuals with high fitness have more chance to be selected for reproduction.

Phases of GA

Crossover

- Child chromosomes are produced by aligning two parents.
- For each pair of parents to be mated, a crossover point is chosen at random from within the genes and swapping the tails with a probability P_c, known as the crossover probability
- The tails of both the chromosomes are swapped to produce a new off-springs.
- Consider an eight-loci chromosome, with the mother and father genes are represented by m_i and f_i respectively.

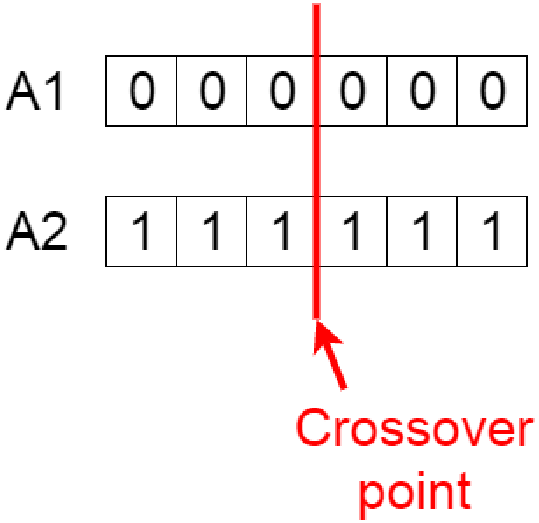
$$\begin{array}{ccccc|ccc} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 \\ f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 \end{array} \rightarrow \begin{array}{cccccccc} m_1 & m_2 & m_3 & m_4 & m_5 & f_6 & f_7 & f_8 \\ f_1 & f_2 & f_3 & f_4 & f_5 & m_6 & m_7 & m_8 \end{array}$$

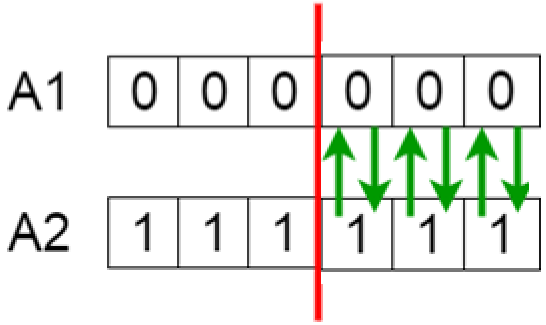
- This is known as *single-point crossover*, as only one position is specified for separating the swapped and unswapped loci.

Phases of GA

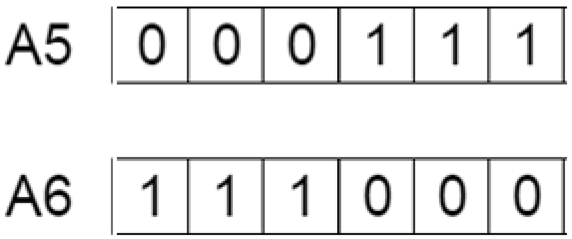
Crossover

- In single-point crossover, the second crossover position is assumed to be the end of the chromosome.





Offspring created by exchanging the genes of parents among themselves after the crossover point.



New offspring are added to the population

Phases of GA

Crossover

- In *two-point crossover*, the chromosomes are treated as though they were circular, that is, m_1 and m_8 are neighboring loci:

$$\begin{array}{c|cccc|ccc} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 \\ & & & & & & & \\ f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 \end{array} \quad \rightarrow \quad \begin{array}{cccccccc} m_1 & f_2 & f_3 & f_4 & f_5 & m_6 & m_7 & m_8 \\ & & & & & & & \\ f_1 & m_2 & m_3 & m_4 & m_5 & f_6 & f_7 & f_8 \end{array}$$

- Multipoint crossover is also possible, provided there is an even number of crossover points

$$\begin{array}{c|cc|cc|cc|c} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 \\ & & & & & & & \\ f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 \end{array} \quad \rightarrow \quad \begin{array}{cccccccc} m_1 & f_2 & f_3 & m_4 & m_5 & f_6 & f_7 & m_8 \\ & & & & & & & \\ f_1 & m_2 & m_3 & f_4 & f_5 & m_6 & m_7 & f_8 \end{array}$$

Phases of GA

Mutation

- In new offspring formed, some of their genes can be subjected to a mutation - involves altering the values of one or more loci.
- Mutation occurs to maintain diversity within the population and prevent premature convergence.
- Mutation can be carried out at either the gene level or the locus level.
- At the gene level, a randomly selected gene can be replaced by a randomly generated allele.
- At the locus level, where values are binary, randomly selected loci can be toggled, that is, 1 becomes 0 and 0 becomes 1.
- Genes or loci are selected randomly for mutation with a probability P_m, that is, the mutation probability.

Before Mutation

A5	1	1	1	0	0	0

After Mutation

A5	1	1	0	1	1	0

Phases of GA

Termination

- The algorithm terminates if the population has converged - does not produce offspring which are significantly different from the previous generation.
- Then it is said that the genetic algorithm has provided a set of solutions to the problem.

Applications of GA

- Engineering design - computer modeling and simulation to make design cycle process fast and economical.
- Traffic and Shipment Routing
- Robotics