Stepper Motors

1. General

Stepper motors are electromagnetic incremental devices that convert electric pulses to shaft motion (rotation). These motors rotate a specific number of degrees as a respond to each input electric pulse. Typical types of stepper motors can rotate 2°, 2.5°, 5°, 7.5°, and 15° per input electrical pulse. Rotor position sensors or sensorless feedback based techniques can be used to regulate the output response according to the input reference command. Stepper motors offers many attractive features such as [1]:

- *Available resolutions ranging from several steps up to 400 steps (or higher) per revolution.*
- *Several horsepower ratings.*
- *Ability to track signals as fast as 1200 pulses per second.*

Stepper motors have many industrial applications such as [1]:

- *Printers.*
- *Disk Drives.*
- *Machine Tools.*
- *Robotics.*
- *Tape Drives.*

2. Types of Stepper Motors

Stepper motors are usually classified into three main categories, namely, V*ariable reluctance (single stack and multi stack)*, *Permanent Magnet*, and *Hybrid motors*.

2.1 Single Stack Variable Reluctance Stepper Motors

Fig. 1 presents the basic circuit configuration of a typical 4-phase, 2-pole, single-stack, variable reluctance stepper motor. The stator is made of a single stack of steel laminations with the phase windings wound around the stator poles. The rotor is made of stack of steel laminations without any windings. The main principle of operation depends on aliging one set only of stator and rotor poles by energizing the stator windings. Therefore, the number of poles in the stator and rotor windings has to be different. The stator windings are energized by a DC source in such a sequence to generate a resultant rotating air-gap field around the rotor in steps. The rotor is made of ferromagnetic material that provides a tendency to align the rotor axis along the direction of the resultant air-gap field. Therefore, the rotor tracks the motion of this stepped field.

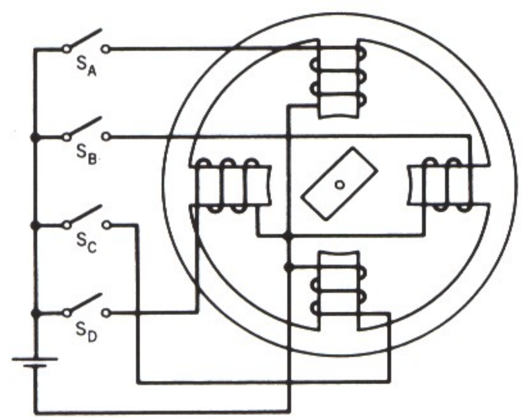

Fig. 1 Basic circuit configuration of a typical 4-phase, 2-pole, single-stack, variable reluctance stepper motor [1]

Fig. 2 illustrates the different modes of operation of the 4-phase, 2-pole, single-stack, variable reluctance stepper motor for 45° step in the following energizing sequence A, A+B, B, B+C, C, C+D, D, and then D+A. Then this switching sequence is repeated.

- ***Energizing winding A:*** The resultant air-gap flux will be aligned along the axis of pole *A* windings. Consequently, the rotor aligns itself along the phase *A* axis as shown in the upper part of Fig. 2.

- ***Energizing windings A and B:*** The resultant air-gap flux will be oriented in the midway between pole *A* and pole *B* i.e., the resultant mmf rotated 45° in the clockwise direction. Consequently, the rotor aligns itself with the resultant mmf (45°) as shown in the middle part of Fig. 2.

- ***Energizing winding B:*** The resultant air-gap flux will be aligned along the axis of pole *B* windings. Consequently, the rotor aligns itself along the phase *B* axis as shown in the lower part of Fig. 2.

- ***and so on.***

The direction of rotation can be reversed by reversing the switching sequence to be A, A+D, D, D+C, C, C+B, B, and then B+A. Then this switching sequence is repeated.

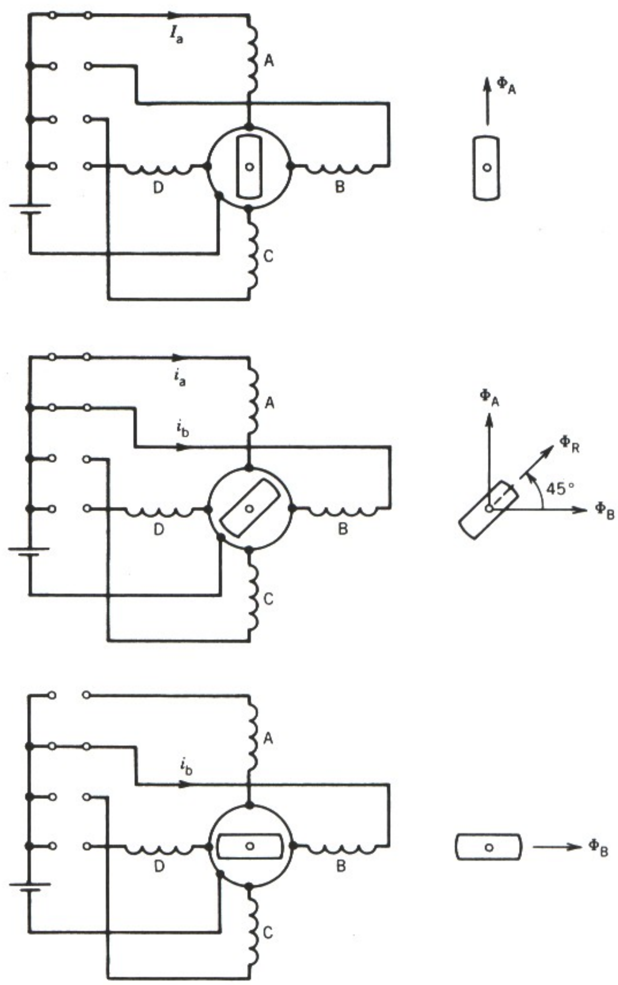

Fig. 2 Operation modes of single-stack, 2-poles, variable reluctance stepper motor with 45° step [1]

Smaller steps can be obtained by using multi-pole rotor configuration such as the one shown in Fig. 3 that rotate in an anticlockwise direction with a 15° step in the following energizing sequence A, A+B, B, B+C, C, C+D, D, and then D+A. Then this switching sequence is repeated.

- ***Energizing winding A:*** The resultant air-gap flux will be aligned along the axis of pole *A* windings. Consequently, the rotor pole P_1 aligns itself along the phase *A* axis as shown in the upper part of Fig. 3.

- ***Energizing windings A and B:*** The resultant air-gap flux will be oriented in the midway between pole *A* and pole *B* i.e., the resultant mmf rotated 45° in the clockwise direction. In this case, the nearest rotor pole to this direction is pole P_2. Consequently, the rotor rotates in an anticlockwise direction to align pole P_2 with the resultant mmf (45°). Therefore, the net rotational step is 15° in an anticlockwise direction.

- ***Energizing winding B:*** The resultant air-gap flux will be aligned along the axis of pole *B* windings. In this case, the nearest rotor pole to this direction is pole P_3. Consequently, the rotor rotates in an anticlockwise direction to align pole P_3 with the resultant mmf (90°). Therefore, the net rotational step in this stage is also 15° in an anticlockwise direction.

- ***and so on.***

The direction of rotation can be reversed by reversing the switching sequence to be A, A+D, D, D+C, C, C+B, B, and then B+A. Then this switching sequence is repeated.

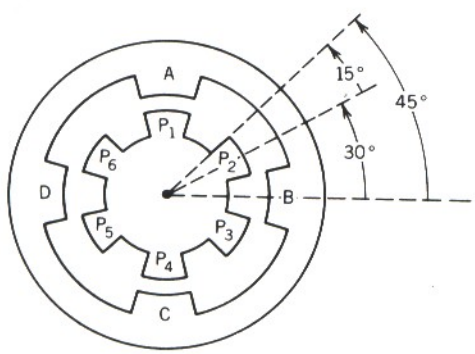

Fig. 3 Construction and operation of 4-phase, 6-pole, single-stack, variable reluctance stepper motor [1]

Fig. 4 presents the circuit configuration and different operation modes for a 3-phase, 4-pole, single-stack, variable reluctance stepper motor that rotate in a clockwise direction with a 30° step. Table 1 and Fig. 5 present each phase switching sequence for one revolution of the rotor.

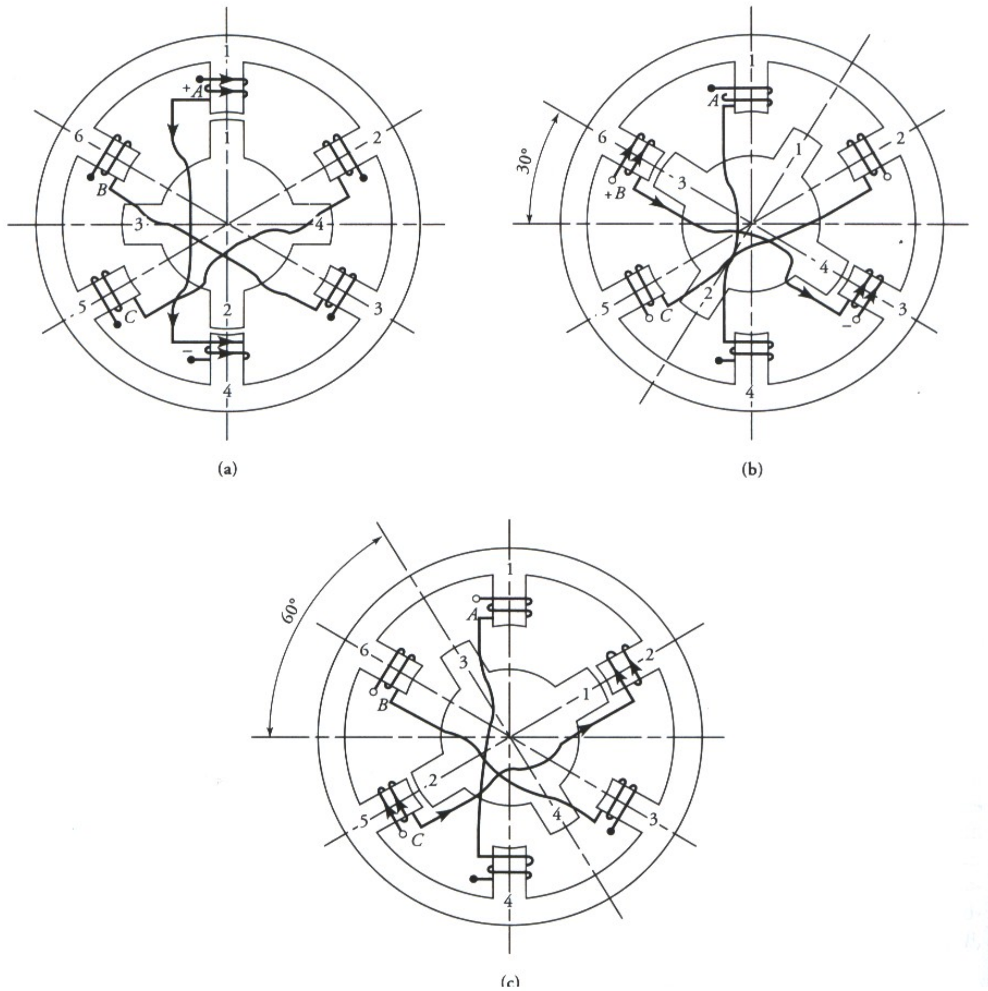

Fig. 4 Construction and operation of 3-phase, 4-pole, single-stack, variable reluctance stepper motor [2]

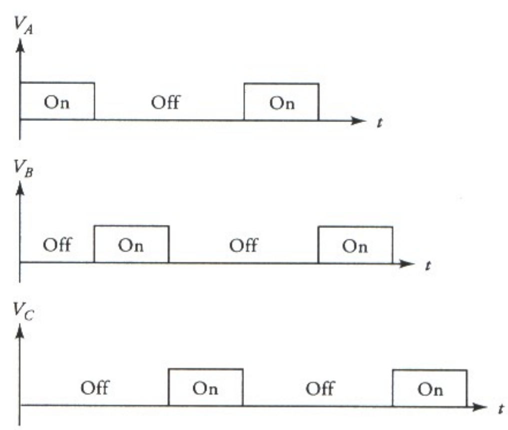

Fig. 5 Phase switching sequence [2]

Table 1 Phase switching sequence: “1” and “0” corresponds to positive and zero phase voltage (currents), respectively [2]

Cycle	Phase A	Phase B	Phase C	Position δ°
1	1	0	0	0
	0	1	0	30
	0	0	1	60
2	1	0	0	90
	0	1	0	120
	0	0	1	150
3	1	0	0	180
	0	1	0	210
	0	0	1	240
4	1	0	0	270
	0	1	0	300
	0	0	1	330
5	1	0	0	360

2.2 Multi-Stack Variable Reluctance Stepper Motors

In this type, the motor is divided along its axis into a number of stacks. Each stack is energized by a separate winding (phase) as shown in Fig. 6. These stacks are magnetically isolated from each other. The most common type is the three-stack, three-phase motors; however, number of stacks and phases up to seven are also available.

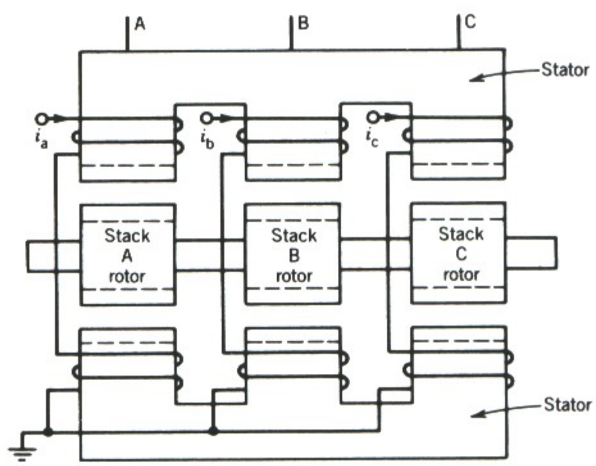

Fig. 6 Cross section view of a typical three-stack variable reluctance stepper motor [1]

These motors are characterized by have the same number of teeth in each of the stator stacks as those in each of the rotor stacks. However, the rotors teeth in the different stacks are aligned while those in the stator stacks are not as shown in Fig. 7.

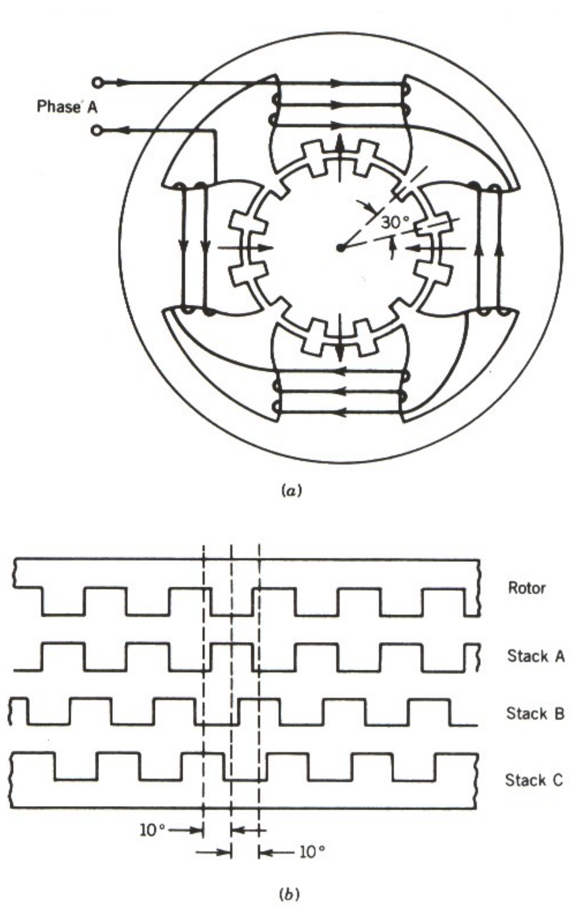

Fig. 7 Teeth position for a 3-phase, 4-pole, 12-teeth, three-stack, variable reluctance stepper motor when phase a is energized [1]

Fig. 7 also illustrates the rotational sequence of a 3-phase, 4-pole, 12-teeth, three-stack, variable reluctance stepper motor for 10° step in a clockwise direction according to the following energizing sequence A, B, and C. Then this switching sequence is repeated.

- ***Energizing phase (stack) A:*** when *stack A* winding is energized, the rotor teeth will move to align themselves with the stator teeth is *stack A* as shown in Fig. 7.

- ***Energizing phase (stack) B:*** when *stack B* winding is then energized while *stack A* winding is de-energized, the rotor teeth will move to align themselves with the stator teeth is *stack B*. This will result in a clockwise rotation of the rotor by 10° as shown in Fig. 7.

- ***Energizing phase (stack) C:*** when *stack C* winding is then energized while *stack B* winding is de-energized, the rotor teeth will move to align themselves with the stator teeth is *stack C*. This will result in another clockwise rotation of the rotor by 10°. After this stage the rotor has moved one rotor tooth pitch as illustrated by Fig. 7.

- ***and so on.***

The direction of rotation can be reversed by reversing the switching sequence. Assume that the total number of stacks (phases) is N while the total number of teeth in each stack is x. The tooth pitch (τ_p) can be expressed by,

$$\tau_p = \frac{360°}{x}$$

Moreover, the step size $(\Delta\theta)$ can be expressed by,

$$\Delta\theta = \frac{360°}{x\ N}$$

Consequently, the number of steps per revolution *(n)* is given by

$$n = \frac{360°}{\Delta\theta} = x\ N$$

As an example, for the motor in Fig. 7, $x = 12$ and $N = 3$. Therefore,

$$\tau_p = \frac{360°}{x} = \frac{360°}{12} = 30° \ ,$$

$$\Delta\theta = \frac{360°}{x\ N} = \frac{360°}{12 * 3} = 10° \ , \qquad \text{and}$$

$$n = \frac{360°}{10} = 36$$

2.3 Permanent Magnet Stepper Motors

Permanent magnet (PM) stepper motors are similar in construction to that of single-stack, variable reluctance stepper motors except that the rotor is made of permanent magnet. Fig. 8 presents the circuit configuration and different operation modes for a 2-phase, permanent magnet stepper motor that rotate in an anticlockwise direction with a 90° step. Table 2 and Fig. 9 present each phase switching sequence for one revolution of the rotor. Reversing the switching sequence will result in reversing the direction of rotation.

PM stepper motors offer many features compared to variable reluctance type such as [1]:

- Higher inertia and consequently lower acceleration (deceleration) rates.
- Maximum step pulse rate is 300 pulses per second compared to 1200 pulses per second for variable reluctance stepper motors.
- Larger step sizes, ranging from 30° to 90° compared to step sizes as low as 1.8° for variable reluctance stepper motors.
- Generate higher torque per ampere of stator currents than variable reluctance stepper motors.

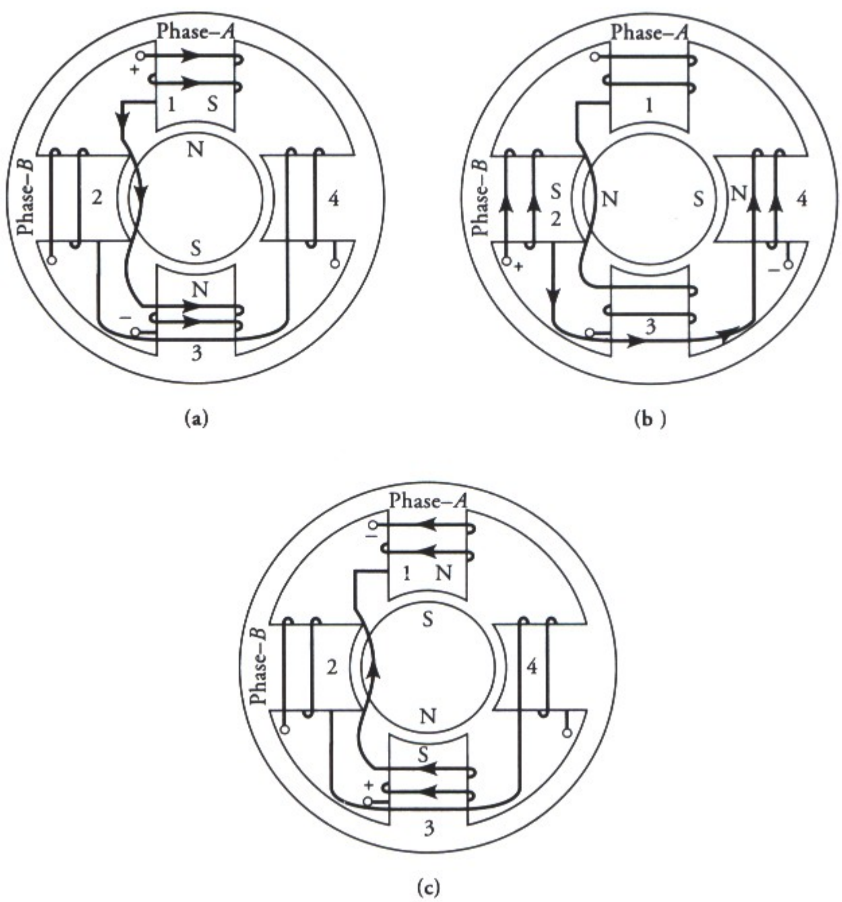

Fig. 8 Construction and operation of 2-phase, permanent magnet stepper motor [2]

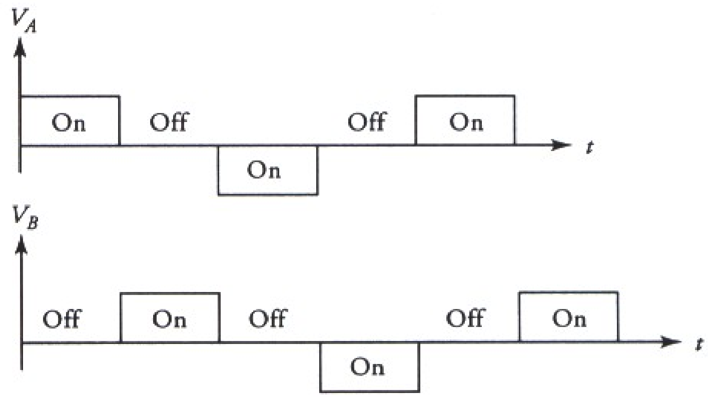

Fig. 9 Phase switching sequence [2]

Table 2 Phase switching sequence: “1”, “-1” and “0” corresponds to positive, negative, and zero phase voltage (currents), respectively [2]

Cycle	Phase A	Phase B	Position δ°
+	1	0	0
	0	1	90
−	−1	0	180
	0	−1	270
+	1	0	360

2.4 Hybrid Stepper Motors

Hybrid stepper motors have similar stators’ construction to that of variable reluctance stepper motors. However, their rotors constructions combine both variable reluctance and permanent magnet constructions. The rotors are made of an axial permanent magnet at the middle and two identical stacks of soft iron poles at the outer ends attached to the north and south poles of the permanent magnet. The rotor poles connected to the north pole of the permanent magnet forms north pole, while the other form the south poles as shown in Fig. 10. This figure also presents two different views of these motors types. Fig. 11 presents a complete cross section view of 4-pole stator and 5-pole rotor hybrid stepper motor while Fig. 12 presents the different components of standard hybrid stepper motor. These types of motors have similar operation modes as the permanent magnet types. Moreover, they are characterized by smaller step sizes but they are very expensive compared to variable reluctance stepper motors.

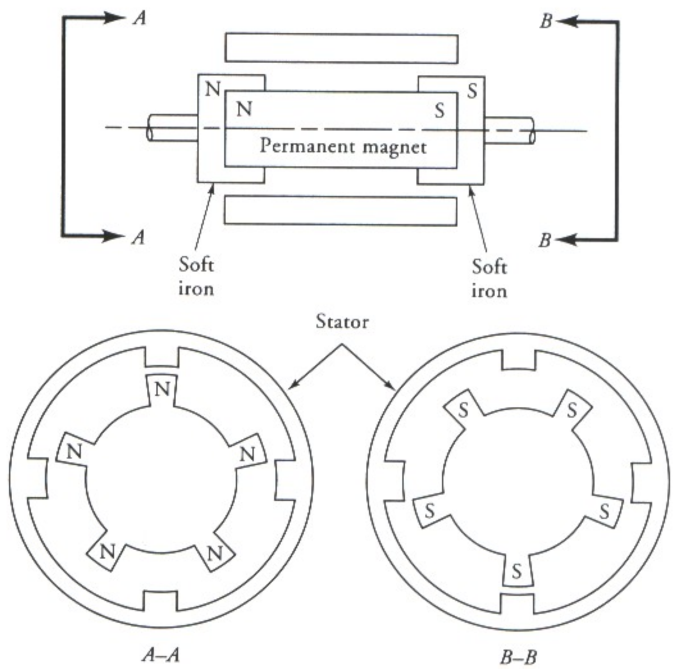

Fig. 10 Construction of 4-pole stator and 5-pole rotor hybrid stepper motor [2]

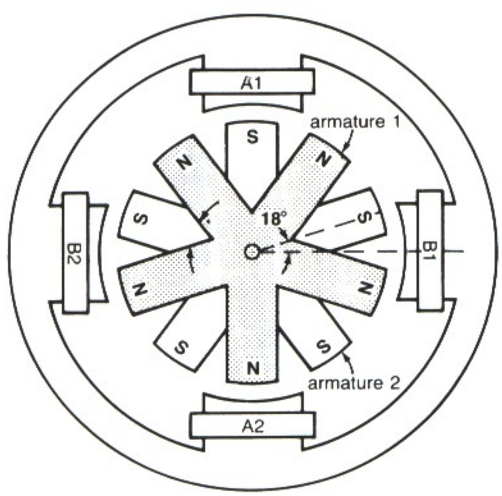

Fig. 11 Cross section view of 4-pole stator and 5-pole rotor hybrid stepper motor [3]

3. Modes of Operation

Consider a 3-phase, 3-poles stepper motor as shown in Fig. 13. When energizing *phase B*, the rotor starts to rotate in an anticlockwise direction to align itself with *pole 2*. It is supposed theoretically, that the rotor will come to rest once its axis is aligned with *pole 2* axis. However, practically, due to the inertia of the rotor, the rotor will overshot and pass the central line of *pole 2*. After that, the magnetic field generated from *pole 2* pulls the rotor in the opposite direction. The rotor will swing around the central line of *pole 2* until finally it comes to rest after being aligned. Fig. 14 a presents the variation of the rotor position and the rotor speed as a result of energizing *phase B*.

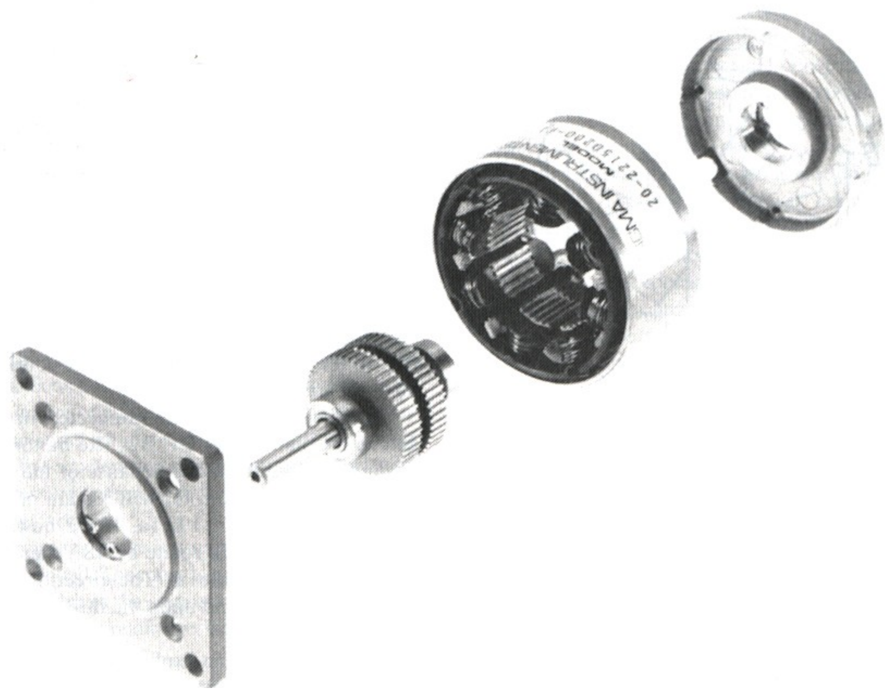

Fig. 12 Different ccomponents of standard hybrid stepper motor [3]

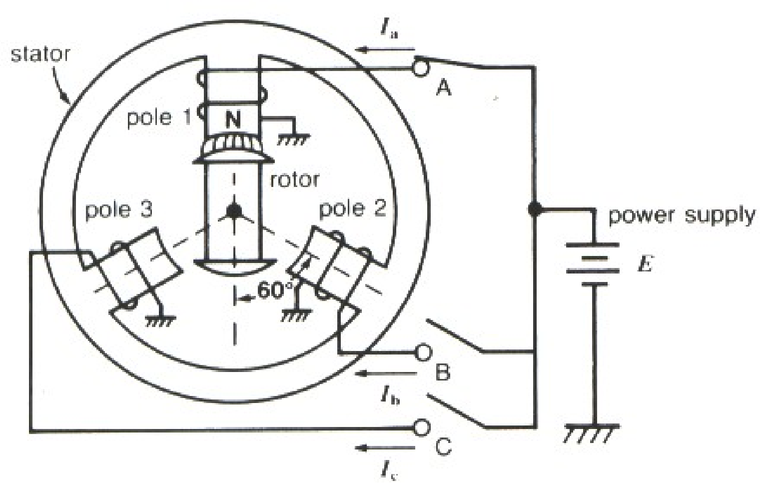

Fig. 13 3-phase, 3-poles stepper motor [3]

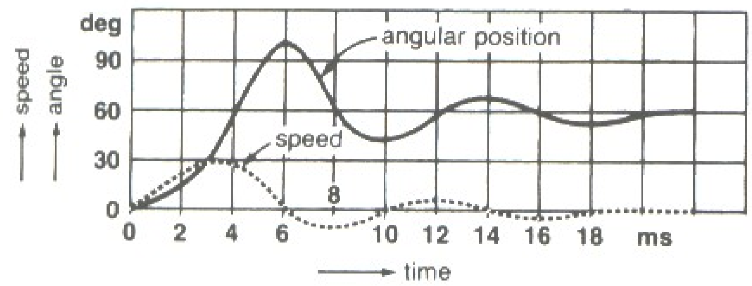

Fig. 14 a Effect of inertia on rotor angular speed and angular position [3]

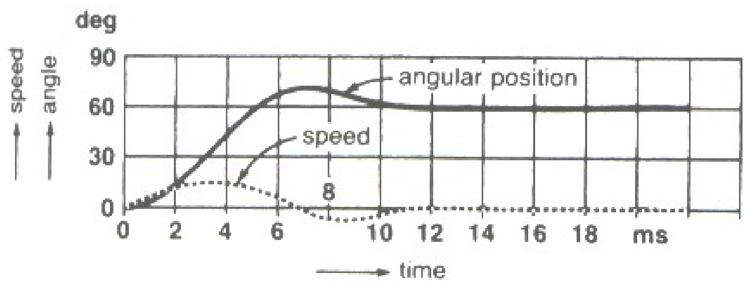

Fig. 14 b Effect of viscous damping on rotor angular speed and angular position [3]

There are two main modes of operation of stepper motors that can be summarized as follows:

- ***Start-Stop Mode:*** In this mode the motor is controlled to settle down (rest) after each step before advancing to the next step. The rotational speed will be in the form of pulses that drops to zero at the end each step while the rotor position will be in the form of pulses also but with an increasing steady state value with time as shown in Fig. 15. This mode is sometimes referred to by the *start without error mode*. A maximum permissible stepping rate is required for this mode of operation; otherwise, the motor will not be able to track the control current pulses and the step will be lost. This minimum rate depends on the motor inertia and the loading condition. Fig. 16 presents the torque-speed (steps per second, where each step equivalent to 1.8°)characteristic for this mode of operation represented by:

    - ***Curve 1:*** Low inertia. If the motor drives a load of 1.4 N.m then the maximum permissible pulse rate is 500 steps per second.

    - ***Curve 2:*** Higher inertia. If the motor drives a load of 1.4 N.m then the maximum permissible pulse rate is 400 steps per second.

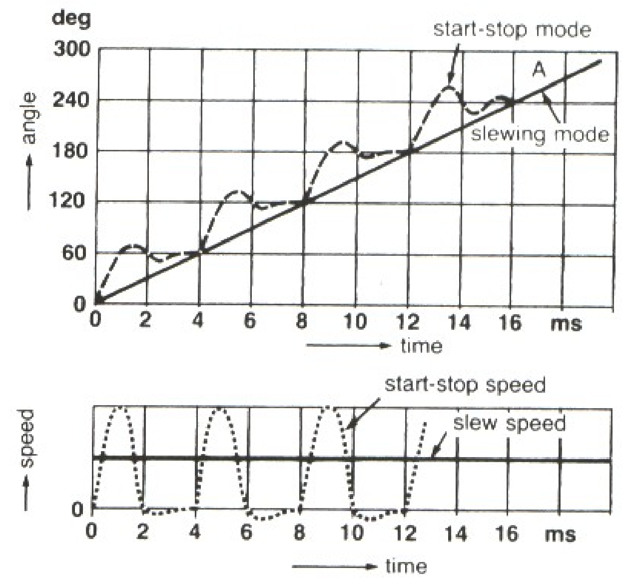

Fig. 15 Rotor angular speed and angular position for different operating modes [3]

- ***Slew Speed Mode:*** In this mode the motor is controlled to rotate at a constant uniform speed without stopping at the end of each step and the rotor position varies linearly with time as shown in Fig. 15. The torque speed characteristic of this mode will not be affected by the system inertia because of the constant speed. Moreover, for a specific pulse rate (500 steps per second) this mode allows the motor to drive higher torque load as in the start-stop mode as shown in Fig. 16.

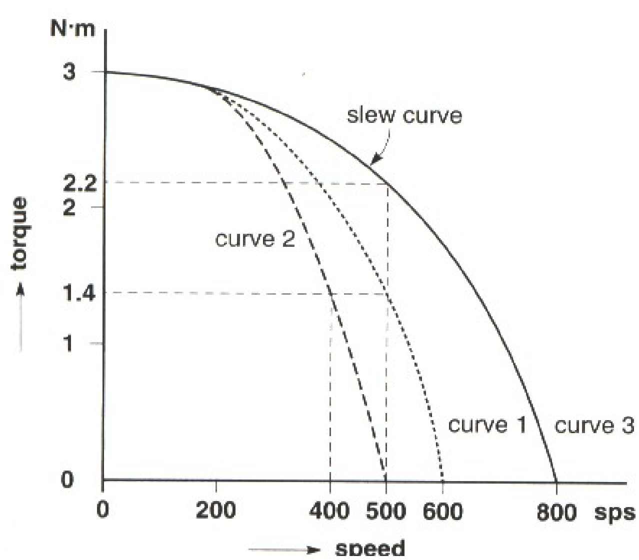

Fig. 16 Torque-speed characteristic [3]

4. Drive Circuit

There are two main drive circuits for stepper motors, namely; *Uni-polar* and *Bi-polar* drive circuits.

4.1 Uni-polar Drive Circuit

Fig. 17 presents a schematic diagram for a uni-polar drive circuit. This circuit is suitable for three-phase variable reluctance stepper motors. Each phase winding of the motor is controlled by a separate drive circuit with a transistor as its controllable power switch. All drive circuits are energized by the same DC source. The transistor (power switch) of each winding has two modes of operation as follows:

- ***On Mode:*** When sufficiently high base current flow through the transistor base it turn ON and acts ideally like a short circuit. Consequently, the supply voltage will be applied across the phase winding and the external resistor (R_{ext}) connected in series with the phase winding. The DC source magnitude is adjusted to produces the rated phase current when the switch is turned ON. Therefore,

  $$V_s = I\left(R_{Ph} + R_{ext}\right)$$

  where V_s is the DC source voltage in *V*, *I* is the phase winding rated current in *A*, R_{ph} is the phase winding resistance in Ω, and R_{ext} is the external resistance connected in series to the phase winding in Ω.

  The phase winding inductance is very large and consequently results in slow rate of building the phase winding current that might result in unsatisfactory operation of the stepper motor at high stepping rates. Therefore, the external resistance is connected in series with the phase winding to reduce the time constant. The net *ON Mode* circuit time constant will be very large and can be expressed by,

  $$\tau_{ON} = \frac{L_{ph}}{\left(R_{Ph} + R_{ext}\right)}$$

  where L_{ph} is the phase winding average inductance in *H*.

- ***OFF Mode:*** In this mode, the base drive current of the transistor is removed and the switch is turned *OFF* and acts as an open circuit. The phase winding current will continue to flow through the freewheeling path formed by the freewheeling diode (D_f)

and the freewheeling resistance (R_f). The maximum *OFF state* voltage appears across the transistor (switch) ($V_{CE\,(max)}$) can be expressed by,

$$V_{CE\,(\max)} = V_s + I\ R_f$$

During this mode of operation, phase current decays in the *OFF mode* circuit with a net *OFF Mode* circuit time constant that can be expressed by,

$$\tau_{OFF} = \frac{L_{ph}}{\left(R_{Ph} + R_{ext} + R_f\right)}$$

The energy stored in the phase inductance during the *ON mode* is dissipated in the OFF mode circuit resistances during the switch turn OFF period.

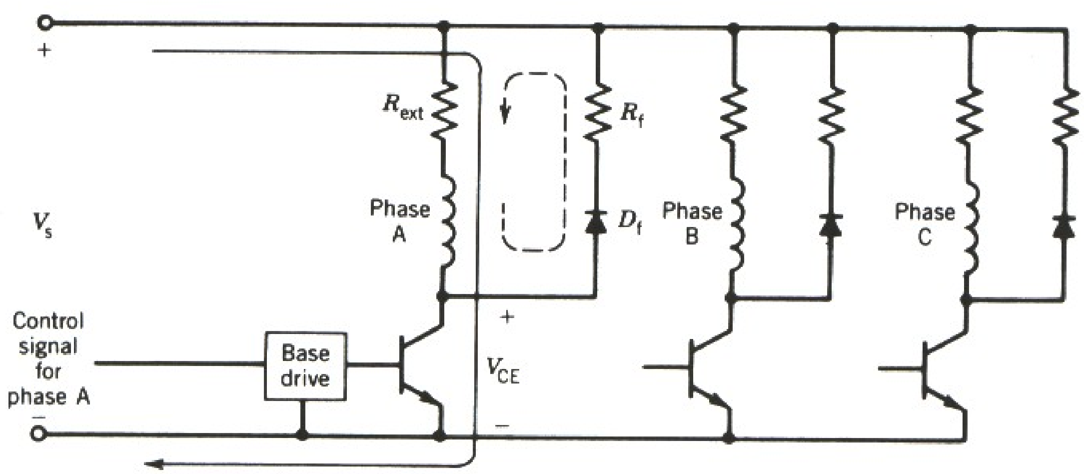

Fig. 17 Uni-polar drive circuit for three-phase variable reluctance stepper motor [1]

4.2 Bi-polar Drive Circuit

Fig. 18 presents a schematic diagram for one phase of a bi-polar drive circuit. This circuit is suitable for permanent magnet or hybrid stepper motors. Each phase winding of the motor is controlled by a separate drive circuit with a transistor as its controllable power switch. All drive circuits are energized by the same DC source. Each two transistors (power switches) of each phase winding are turned ON simultaneously. Two modes of operation occur as follows:

- ***T_1 and T_2 are in the On Mode:*** This is done by injecting sufficiently high base current through their bases simultaneously. Each transistor acts ideally like a short circuit.

Consequently, the current will flow as indicated by the solid line in Fig. 18. The inductor is then energized.

- ***D_3 and D_4 are in the On Mode:*** This mode follows the switching *OFF* of T_1 and T_2. In this mode, the phase winding current cannot change its direction or decay to zero instantaneously after turning *OFF* of T_1 and T_2 because of the phase winding inductances. Thus the current continue to flow through of D_3 and D_4 as indicated by the dotted line in Fig. 18. The inductor discharges and the energy is returned back to the DC source.

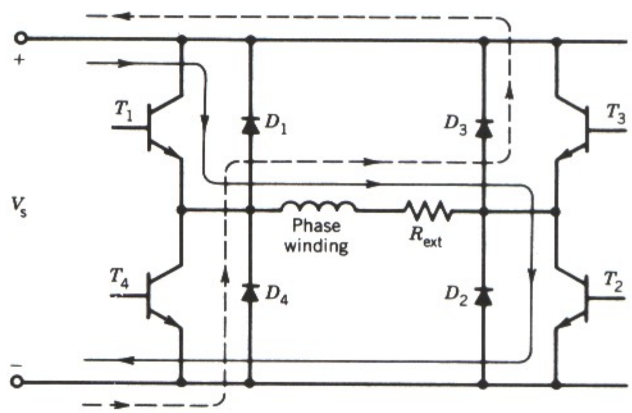

Fig. 18 One phase of a Bi-polar drive circuit for permanent magnet or hybrid stepper motors [1]

A reverse flow of current in the phase windings and hence a reverse direction of rotation of the motor can be achieved by activating T_3 and T_4. When T_3 and T_4 are turned *OFF* the freewheeling path will provided through D_1 and D_2. The bi-polar circuit is characterized by,

- Higher efficiency than the uni-polar drive circuit as part of the stored energy in the phase winding returns back to the DC source during the power switches turn OFF mode.

- Fast decaying of the freewheeling current as the inductor discharge through the phase winding resistance, phase external resistance and the DC source.

- No freewheeling resistance is required.

- More power switches (devices) than the uni-polar drive circuit.

- More expensive than the uni-polar drive circuit.
- Most of the large stepper motors types (> 1 kW) are driven by the bi-polar drive circuit including variable reluctance types.

5. Switching Sequence

Consider the 4-pole hybrid stepper motor shown in Fig. 11. The motor is drived by a Bi-polar drive circuit where the power switches are represented by contacts as illustrated by Fig. 19. Four contacts are used with each coil set of the motor (*A1* and *A2*) and (*B1* and *B2*). The two coil sets are energized by the same DC source. There are three main switching techniques for controlling these contacts namely; *Wave Switching*, *Normal Switching*, and *Half-Step Switching*.

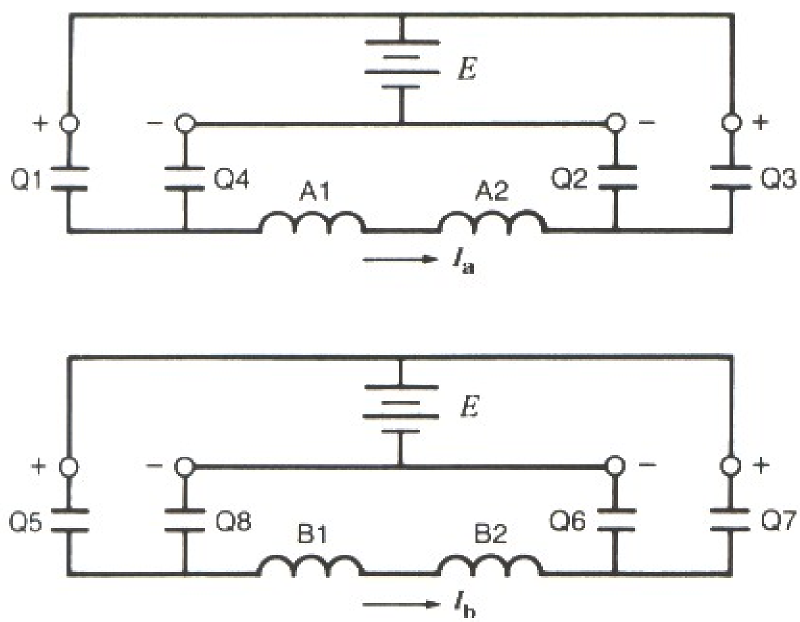

Fig. 19 Drive circuit for the hybrid motor under consideration [3]

- **Wave Switching Sequence:** In this technique, only one set of coils is switches each step and the generated flux rotates by 90° per step. Table 3 presents the switching sequence for clockwise rotation. The corresponding coils' current pulses and the generated fluxes are shown in Fig. 20.

Table 3 Wave switching sequence for clockwise rotation [3]

WAVE SWITCHING SEQUENCE FOR CW ROTATION						
Step		1	2	3	4	1
Q1	Q2	on	—	—	—	on
Q3	Q4	—	—	on	—	—
Q5	Q6	—	on	—	—	—
Q7	Q8	—	—	—	on	—

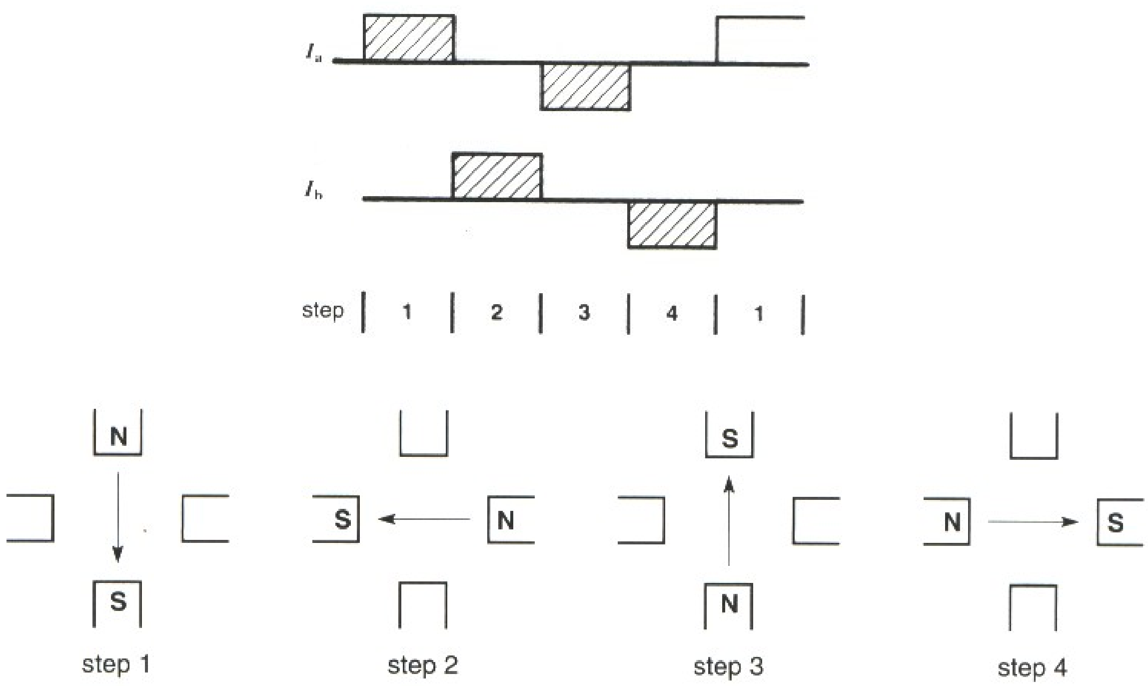

Fig. 20 Current pulses and generated fluxes for wave switching sequence [3]

- **Normal Switching Sequence:** In this technique, the two sets of coils are switches each step. The generated flux also rotates by 90° per step; however, it is oriented in the midway between the stator's poles. Table 4 presents the switching sequence for clockwise rotation. The corresponding coils' current pulses and the generated fluxes are shown in Fig. 21. This technique is characterized by slightly greater torque than the wave switching sequence.

Table 4 Normal switching sequence for clockwise rotation [3]

TABLE 19B NORMAL SWITCHING SEQUENCE FOR CW ROTATION

Step		1	2	3	4	1
Q1	Q2	on	—	—	on	on
Q3	Q4	—	on	on	—	—
Q5	Q6	on	on	—	—	on
Q7	Q8	—	—	on	on	—

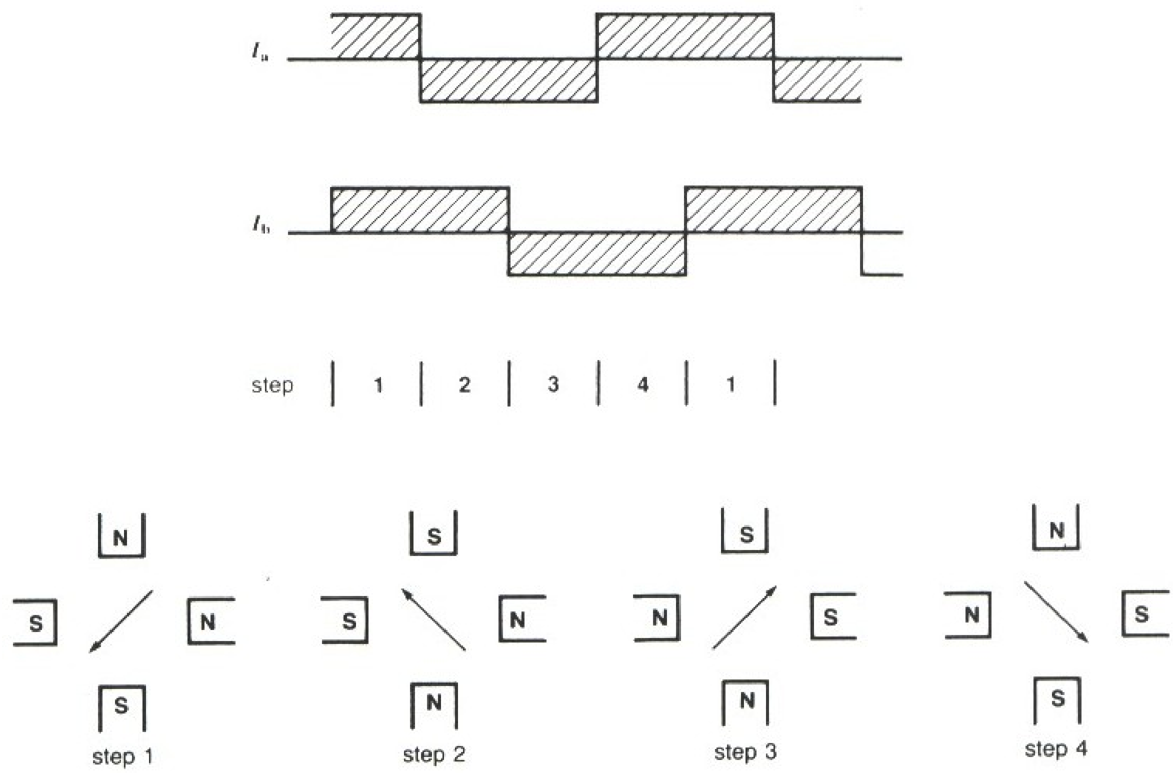

Fig. 21 Current pulses and generated fluxes for normal switching sequence [3]

- **Half-Step Switching Sequence:** In this technique, both the wave and the normal switching sequence are combined. The generated flux also rotates by 45° per step. Table 5 presents the switching sequence for clockwise rotation. The corresponding coils' current pulses and the generated fluxes are shown in Fig. 22. This technique is characterized by better resolution of position and reduction in the resonance problem.

Table 5 Half-step switching sequence for clockwise rotation [3]

TABLE 19C HALF-STEP SWITCHING SEQUENCE FOR CW ROTATION

Step		1	2	3	4	5	6	7	8	1
Q1	Q2	on	on	—	—	—	—	—	on	on
Q3	Q4	—	—	—	on	on	on	—	—	—
Q5	Q6	—	on	on	on	—	—	—	—	—
Q7	Q8	—	—	—	—	—	on	on	on	—

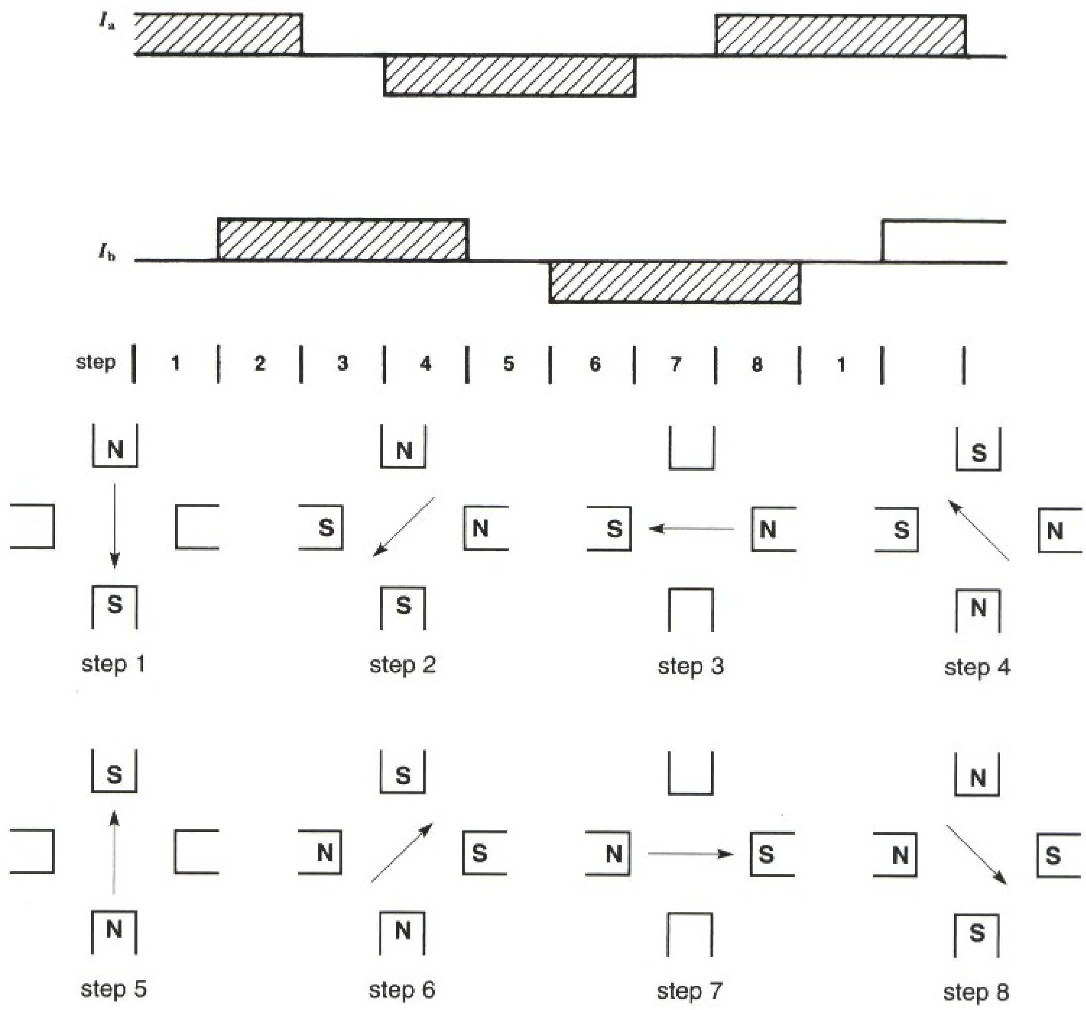

Fig. 22 Current pulses and generated fluxes for half-step switching sequence [3]

6. High Speed Operation

In the previous analysis, the current waveforms is considered to be in the form of rectangular pulses (ideal case) that reach it is peak value and drops from peak to zero in no time as shown in Fig. 23-a.

However, in practice, because of the system inductance, the current can not change its value instantaneously. This can be explained by considering the circuit shown in Figs. 23-b and 23-c. In this configuration, an inductive load is connected to a DC source via transistor. A freewheeling diode is also used to provide a continuous path for the load current when the transistor is switched *OFF*. Fig. 23-d presents the shape of the current pulse in this case. There are two operation modes for this circuit that can be explained as follows:

- ***Mode 1 (Transistor is ON):*** The transistor behaves as short circuit. The load (motor's phase) current ($i = i_1$) starts to flow with an increasing magnitude until it reaches its rated value I (where $I = \frac{E}{R}$) in approximately 3 times the circuit time constant ($\tau_o = \frac{L}{R}$) i.e. time to reach rated current is $3 \; x \; \tau_o = \frac{3\,L}{R}$. The load current continue to flow with its rated value ($i = I$) until the transistor is switched *OFF* at $t = \tau_p$.

- ***Mode 2 (Transistor is OFF):*** The transistor behaves as open circuit. The load current ($i = i_2$) will continue to circuit in the freewheeling circuit path as shown in Fig. 23-c. This current starts to decay until it reaches zero in approximately 3 times the circuit time constant i.e. time to decay to zero is also $3 \; x \; \tau_o = \frac{3\,L}{R}$.

This practical pulse shape is characterized by:

- The total current period is $\tau_p + 3\,\tau_o$. This results in delaying the switching *ON* process of the next phase in stepper motor.

- The initial torque developed by stepper motors is less than its ideal value because the current doesn't reach its rated value instantly.

- The short current pulse period required to allow the current to reach its rated value is approximately 6 times the circuit time constant ($6 \; x \; \tau_o = \frac{6\,L}{R}$) as shown in Fig. 23-e. Normally, stepper motors have time constants ranging from 1 to 8 ms. Therefore, the minimum permissible pulse period (minimum duration for one step) is 6 ms which corresponds to a maximum stepping rate of about 166 steps per second.

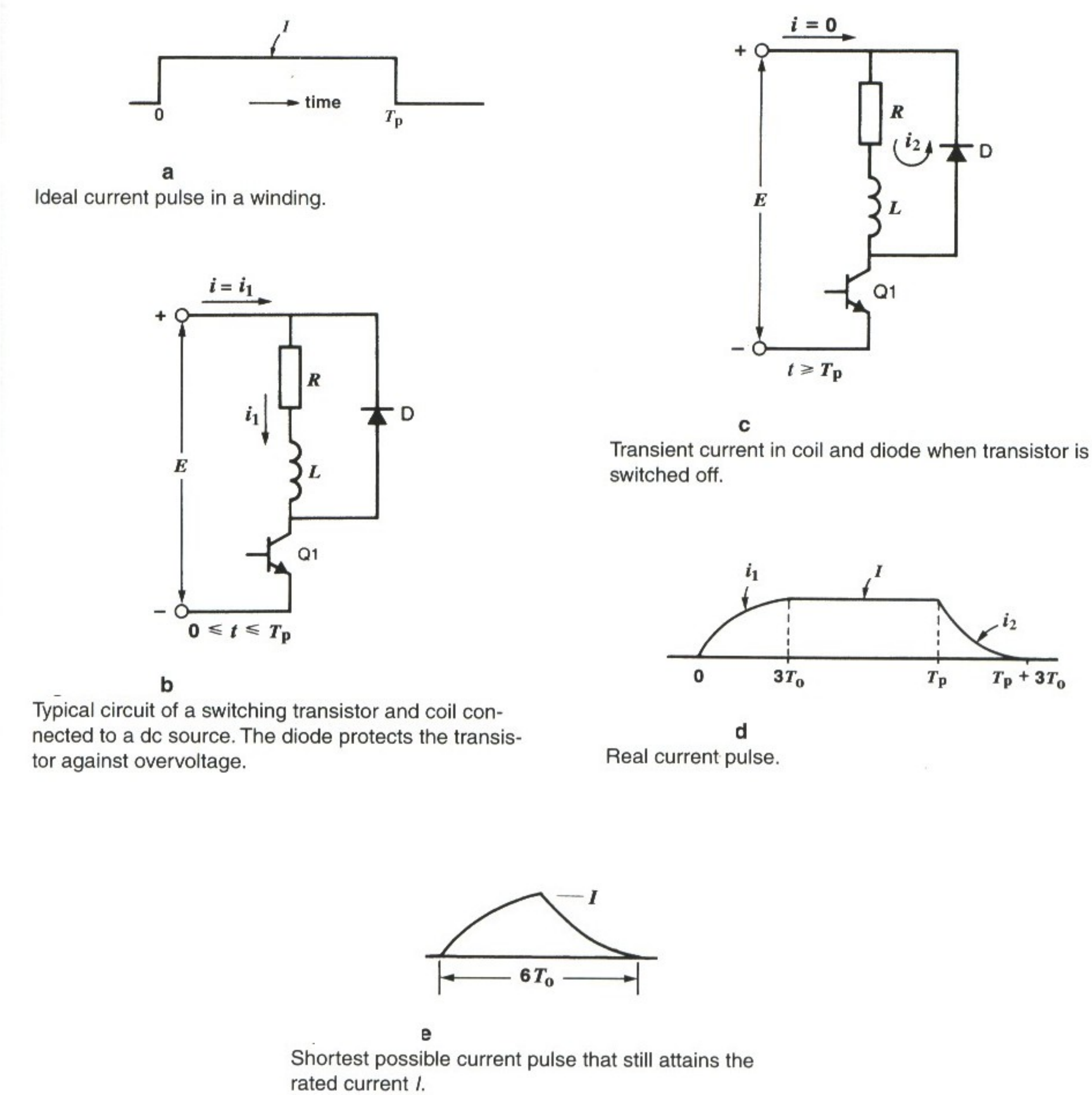

Fig. 23 High speed operation equivalent circuits and current waveforms [3]

There two methods that can be applied to increase the switching rate. These methods can be explained briefly as follows:

- ***External Resistance:*** In this method, an external resistance is inserted in series with the motor's phase winding, as shown in Fig. 24, to reduce the circuit time constant. This figure shows that an external resistance with a resistive value four times the phase winding resistance is added. In this case, the new time constant is $\tau_o = \dfrac{L}{5R}$.

Consequently, the new minimum permissible pulse period is 1.2 ms which corresponds to a maximum stepping rate of about 833 steps per second. However, this method requires increasing the DC source rating to five time its initial vale *5E* as it has to deliver five times the original power. Moreover, the power losses in the resistances are very high.

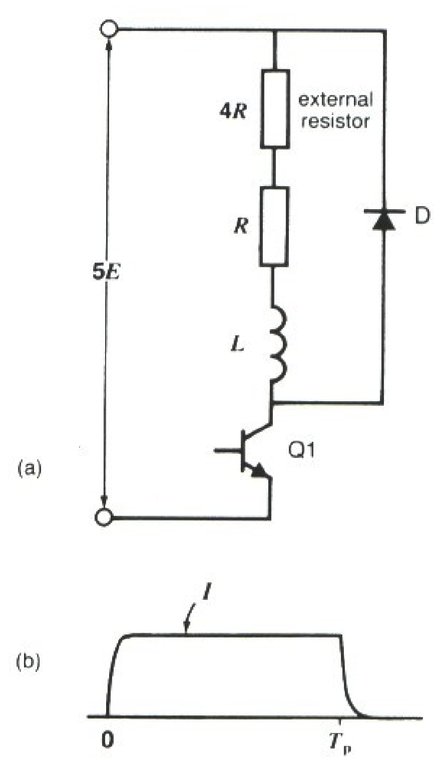

Fig. 24 Circuits to modify the rise and fall time of the current pulse and the associated current [3]

- ***Bi-level Drive:*** In this method, two controllable switches (transistors) and two diodes are used in each phase as shown in Fig. 25-a. Moreover, an additional DC source (E_{ext}) with higher magnitude than the original source (E) is also used. This circuit has three modes of operation that can be explained briefly, with numerical values, as follows:
    - ***Q1 and Q2 are switched ON:*** This is represented by the equivalent circuit shown in Fig. 25-b. The transistors behave as short circuits.

The load (motor's phase) current starts to flow with an increasing magnitude until it reaches its rated value of,

$$I = \frac{E}{R} = \frac{3}{0.3} = 10 \quad A$$

The circuit time constant is given by,

$$\tau_o = \frac{L}{R} = \frac{2.4\ x\ 10^{-3}}{0.3} = 8 \quad ms$$

Therefore the current approximately increases linearly with an increasing rate ($Rate_1$) as shown in Fig. 25-c that can be expressed by,

$$Rate_1 = \frac{\frac{E + E_{ext}}{R}}{\tau_o} = \frac{\frac{3+57}{0.3}}{0.008} = 25{,}000 \quad A/s$$

The time required to reach the phase winding's rated current (t_1) is therefore approximated to,

$$t_1 = \frac{I}{Rate_1} = \frac{10}{25{,}000} = 0.4 \quad ms.$$

- ***Q1 is switched OFF while Q2 is still ON:*** This mode is activated once the phase current reaches its rated value of 10 A. The switch *Q1* is switched *OFF* while the switch *Q2* remains conducting. This is represented by the equivalent circuit shown in Fig. 25-d. The switch *Q1* behaves as an open circuit. In this case the diode *D1* will conduct and the current flows as shown in Fig. 25-d with a constant magnitude of *10 A* ($I = \frac{E}{R} = \frac{3}{0.3} = 10 \quad A$).

- ***Q1 and Q2 are switched OFF:*** The current will remain flowing in the phase winding circuit until switch *Q2* is turned *OFF*. Both switches are now in their *OFF* state and behave as open circuits. In this case the two diodes *D1* and *D2* will conduct and the current flows as shown in Fig. 25-e.

  Assume that the switch *Q2* is turned *OFF* after 5 ms from the instant at which the current reached its rated value. The circuit time constant is given by,

$$\tau_o = \frac{L}{R} = \frac{2.4\ x\ 10^{-3}}{0.3} = 8 \quad ms$$

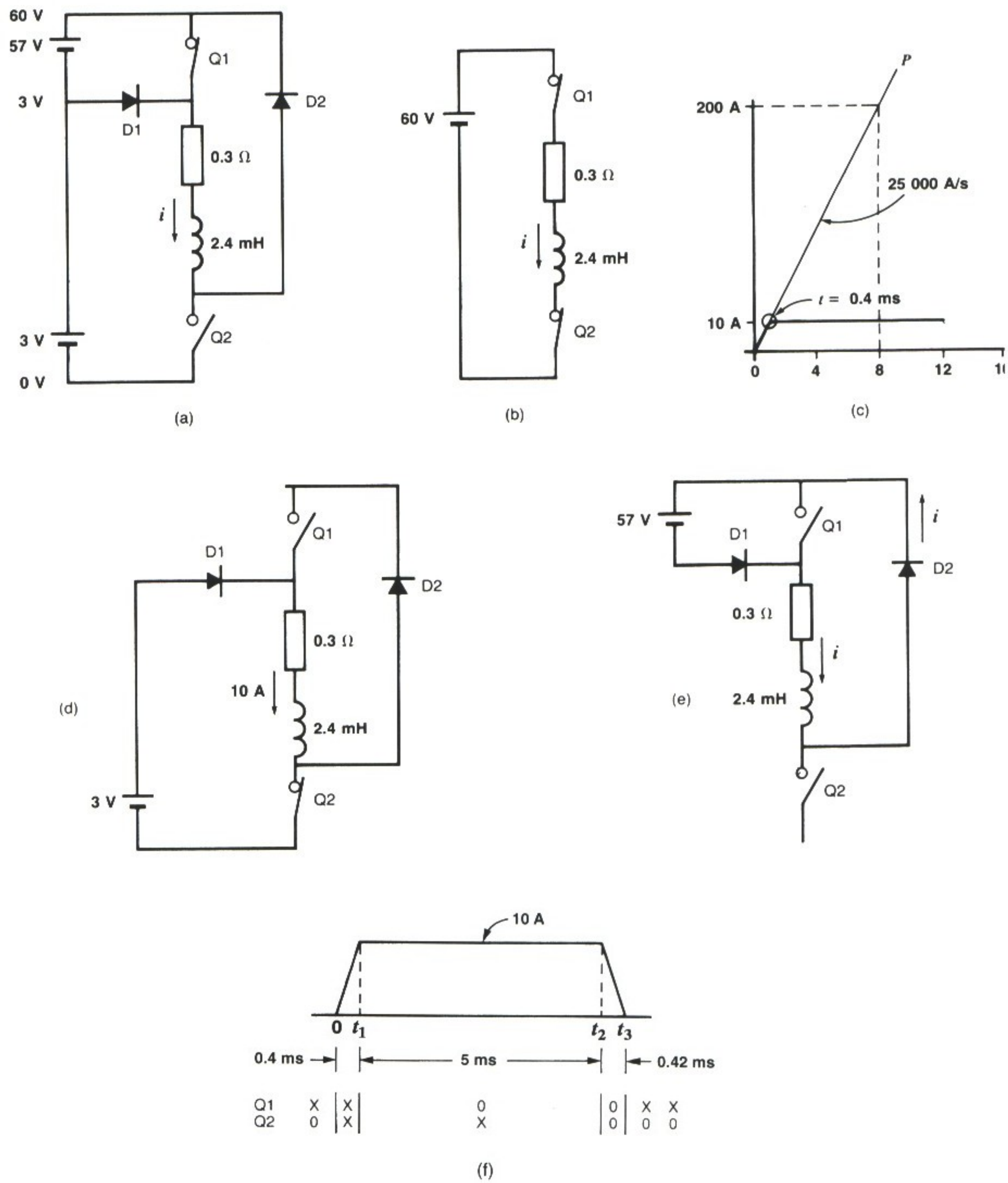

Fig. 25 Bi-level drive circuits to modify the rise and fall time of the current pulse and the associated current [3]

Therefore the current approximately decreases linearly with an decreasing rate ($Rate_2$) as shown in Fig. 25-f that can be expressed by,

$$Rate_2 = \frac{\frac{E_{ext}}{R}}{\tau_o} = \frac{\frac{57}{0.3}}{0.008} = 23{,}750 \quad A/s$$

The time required for the current to decay to zero (t_2) is therefore approximated to,

$$t_2 = \frac{I}{Rate_2} = \frac{10}{23{,}750} = 0.42 \quad ms.$$

Once the current reaches zero, switch *Q1* is switched *ON* to force the phase current to remain zero until the next pulse.

7. Numerical Examples

Example 1: A three-phase, variable reluctance stepper motor has a phase winding resistance and average inductance of 1 Ω and 30 mH, respectively. The phase winding rated current is desired to be 3 A. Design a uni-polar drive circuit for this motor with a net *ON Mode* and *OFF Mode* circuit time constants of 2 msec and 1 msec, respectively. Assume that the stepping rate is 300 steps per second.

Given: I = 3 A, L_{ph} = 30 mH, R_{ph} = 1 Ω, Stepping rate = 300 steps / sec., τ_{ON} = 1 msec and τ_{OFF} = 1 msec.

Solution: For the uni-polar drive circuit shown in Fig. 26.

The net *ON Mode* circuit time constant can be expressed by,

$$\tau_{ON} = \frac{L_{ph}}{\left(R_{Ph} + R_{ext}\right)} = 0.002$$

$$\therefore \left(R_{Ph} + R_{ext}\right) = \frac{L_{ph}}{\tau_{ON}} = \frac{0.030}{0.002} = 15 \quad \Omega$$

$$\therefore R_{ext} = 15 - R_{ph} = 15 - 1 = 14 \quad \Omega$$

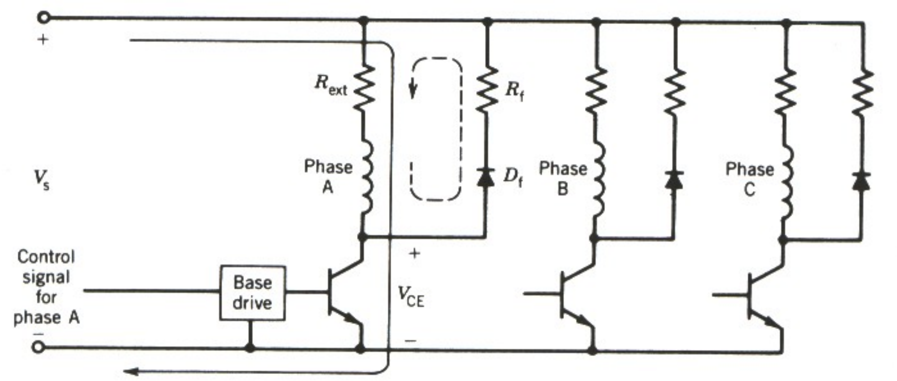

Fig. 26 Uni-polar drive circuit for three-phase variable reluctance stepper motor [1]

This resistance has to be rated to dissipate the power when the rated current flow through it. Therefore,

$$P_{R_{ext}} = I^2 \left(R_{ext}\right) = (3)^2 \ x\, 14 = 126 \quad W$$

The required DC source is,

$$V_s = I\left(R_{Ph} + R_{ext}\right) = 3 \ x\, 15 = 45 \quad V$$

The net *OFF Mode* circuit time constant that can be expressed by,

$$\tau_{OFF} = \frac{L_{ph}}{\left(R_{Ph} + R_{ext} + R_f\right)}$$

$$\therefore \left(R_{Ph} + R_{ext} + R_f\right) = \frac{L_{ph}}{\tau_{OFF}} = \frac{0.030}{0.001} = 30 \quad \Omega$$

$$\therefore R_f = 15 - R_{ph} - R_{ext} = 30 - 15 = 15 \quad \Omega$$

The energy stored in the phase winding is

$$E = \frac{1}{2} I^2 \ L_{ph} = \frac{1}{2} \ x\, (3)^2 \ x\, 0.03 = 0.135 \quad J$$

This energy is dissipated in the OFF mode circuit resistances $\left(R_{Ph} + R_{ext} + R_f\right)$. Since $R_{Ph} + R_{ext} = R_f = 15 \quad \Omega$ Therefore, the energy dissipated in the freewheeling resistance is, *E = 0.5 x 0.135 = 0.0675 J*.

Since the total stepping rate is 300 steps / sec., then the number of turnoffs per phase is 300 / 3 is 100. Therefore,

Average power dissipated in R_f *= 100 x 0.0675 = 6.75 W*.

For the freewheeling diode design: The peak current is the rated current $I = 3\ A$ and the peak reverse voltage (when the transistor is ON) is $V_s = 45\ V$.

For the transistor design: The peak current is the rated current $I = 3\ A$ and the peak OFF state voltage (when the transistor is OFF) is,

$$V_{CE\,(\max)} = V_s + I\ R_f = 45 + 3\ x\ 15 = 90 \quad V$$

Example 2: A stepper motor has a phase winding total resistance and average inductance of 15 Ω and 30 mH, respectively. The phase winding rated current is desired to be 3 A. The motor is driven by a bi-polar drive circuit energized from a DC supply of 45 V. When the transistors are turned OFF, determine

1. The time taken by the phase winding current to decay to zero,
2. The percentage of the stored inductive energy returned to the DC source.

Given: I = 3 A, L_{ph} = 30 mH, R = 15 Ω, and V_s = 45 V.

Solution: For the bi-polar drive circuit shown in Fig. 27. The equivalent circuit at turn OFF mode is shown in Fig. 28.

1. The net *OFF Mode* current can be considered consisting of two components as shown in Fig. 28. The decayed component (i_1) that can be expressed by,

$$i_1 = I\ e^{-\frac{t}{\tau_{OFF}}}$$

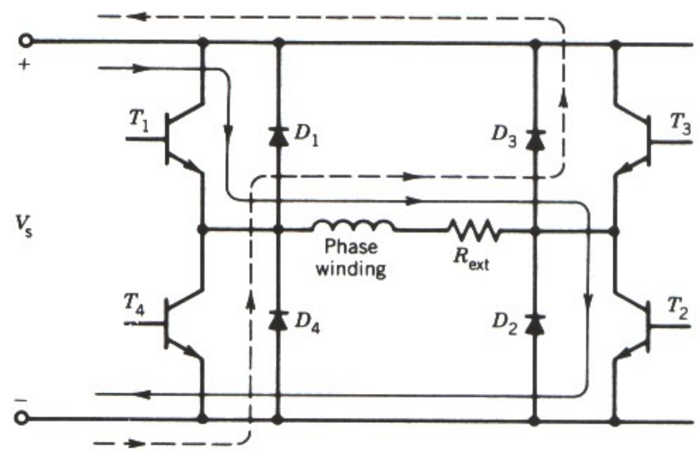

Fig. 27 Bi-polar drive circuit for stepper motor [1]

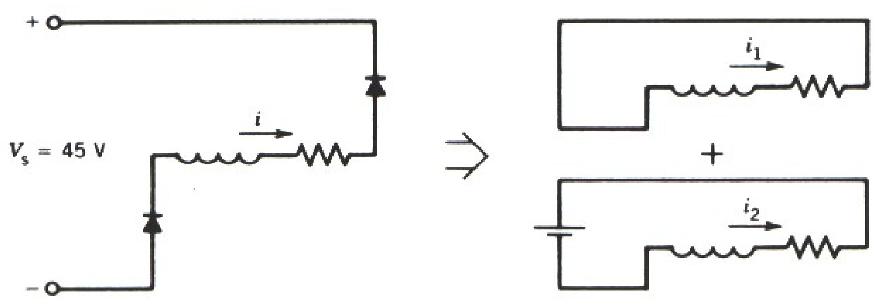

Fig. 28 Equivalent circuit at turn OFF mode operation [1]

where the *OFF* mode time constant is given by,

$$\tau_{OFF} = \frac{L_{ph}}{R} = \frac{0.03}{15} = 0.002 \quad \text{sec.} = 2 \quad m\,\text{sec.}$$

$$\therefore i_1 = 3\ x\ e^{-\frac{t}{0.002}} = 3\ x\ e^{-500\,t}$$

The second components is the source current component (i_2) that can be expressed by,

$$i_2 = -I\left(1 - e^{-\frac{t}{\tau_{OFF}}}\right) = -3\ x\ \left(1 - e^{-500\,t}\right)$$

The negative sign because the current is assumed flowing in the opposite direction.

Therefore, the net *OFF Mode* current can be expressed by,

$$i = i_1 + i_2 = 3\ x\ e^{-500\,t} - 3\ x\ \left(1 - e^{-500\,t}\right)$$

$$\therefore i = 6 \; x \; e^{-500\,t} - 3$$

For this net current to decay to zero, the required time (t_1) is,

$$i = 0 = 6 \; x \; e^{-500\,t_1} - 3$$

$$\therefore \frac{6}{3} = 2 = e^{500\,t_1}$$

$$\therefore t_1 = 0.00139 \quad \sec. = 1.39 \quad m\sec.$$

2. The returned energy to the DC source (E_s) is,

$$E_s = \int_0^{t_1} V_s \cdot i \; dt$$

$$\therefore E_s = \int_0^{t_1} 45 \left(6 \; x \; e^{-500\,t} - 3\right) dt$$

$$\therefore E_s = \int_0^{t_1} -135 \; dt + \int_0^{t_1} 270 \; x \; e^{-500\,t} \; dt$$

$$\therefore E_s = -135 \; t\Big|_0^{t_1} - \frac{270}{500} x \; e^{-500\,t}\Big|_0^{t_1}$$

$$\therefore E_s = -135 \, t_1 - 0.54\left(e^{-500\,t} - 1\right)$$

$$\therefore E_s = -135 \; x \; 0.00139 - 0.54 \left(e^{-500 \; x \; 0.00139} - 1\right) = 0.08285 \quad J = 82.85 \quad mJ$$

The stored energy in the inductor (E_L) is,

$$E_L = \frac{1}{2} I^2 \; L_{ph} = \frac{1}{2} \; x \; (3)^2 \; x \; 0.03 = 0.135 \quad J = 135 \quad mJ$$

Therefore, the percentage of the stored inductive energy returned to the DC source ($\% \; E_s$) is

$$\% \; E_s = \frac{E_s}{E_L} \; x \, 100 = \frac{82.85}{135} \; x \, 100 = 61.37 \quad \%$$

References

[1] P.C. Sen, "Principles of Electric Machines and Power Electronics," Second Edition, John Wiley & Sons, USA, 1997.

[2] Guru Hiziroglu, "Electric Machinery and Transformers," Third Edition, Oxford University Press, USA, 2001.

[3] Theodore Wildi, "Electrical Machines Drives, and Power Systems," Prentice Hall, Ohio, 2006.